What are the plane waves?

Waves? means for transferring energy or information from one place to another.

EM waves: it means for transferring electromagnetic energy

plane waves? mathematically assumes the following form.

$$\vec{F}(\vec{r},t) = \vec{F_0}\, e^{j(\vec{k}\cdot\vec{r}-\omega t)}$$

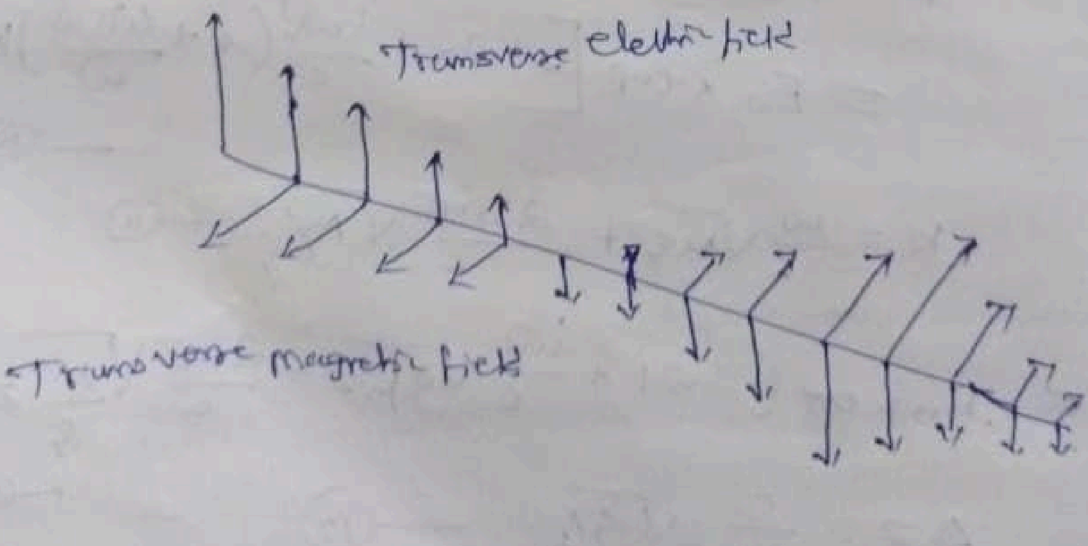
$\vec{k}$ is the wave vector and it points in the direction of wave propagation.

$\vec{r}$ is the general position vector.

ω is the angular frequency and $\vec{F_0}$ is a constant vector

$\vec{F_0}$ denotes either an electric (or) magnetic field (F is a notation for field not for the force)

$\vec{F_0}$ is either vector electric ($\vec{E_0}$ or) magnetic field ($\vec{H_0}$)



(15) and the transmitted wave (E', B') travelling in the Z-direction. The interface is taken as coincident with the x-y plane at $z=0$, with two dielectric media with the indices of refraction, n for $z<0$ and n' for $z>0$, the electric fields, which are assumed to be linearly polarized in the x-direction, are described by.

$$\left.\begin{aligned} E &= e_x E e^{i(kz-\omega t)} \\ E' &= e_x E' e^{i(k'z-\omega t)} \\ E'' &= -e_x E'' e^{i(-kz-\omega t)} \end{aligned}\right\} \quad -①$$

where

$$k = n\omega/c, \quad k' = n'\omega/c,$$

$$B = \frac{n}{ck} k \times E$$

Therefore, the magnetic fields associated with the electric fields of eqn ① are given by,

(24) $$\frac{1}{\mu}\nabla\times B = \frac{1}{c}\left(\epsilon + \frac{4\pi i\sigma}{\omega}\right)\frac{\partial E}{\partial t} \quad \text{—(8)}$$

The inclusion of the conduction current in maxwell's eqn. has the same effect as replacing the dielectric constant ϵ by $\epsilon + 4\pi i\sigma/\omega$.

conducting medium as having a complex index of refraction

$$n = \sqrt{\left(\epsilon + \frac{4\pi i\sigma}{\omega}\right)\mu} \quad \text{—(9)}$$

The wave vector is

$$k = \frac{n\omega}{c} = \omega/c\sqrt{\left(\epsilon + \frac{4\pi i\sigma}{\omega}\right)\mu}$$

and a plane wave travelling in the z direction with a polarization in the x direction has the form

$$E = E_0\hat{x}e^{-i(\omega - kz)}$$

$$= E_0\hat{x}\exp\left[-i\omega x + \frac{i\omega}{c}\sqrt{\left(\epsilon + \frac{4\pi i\sigma}{\omega}\right)\mu}\, z\right] \quad \text{—(10)}$$

$$k = \frac{\omega}{c}\sqrt{\mu\epsilon} + \frac{2\pi i\sigma}{c}\sqrt{\mu/\epsilon} \quad \text{—(11)}$$

$$E = E_0\hat{x}\exp\left(-i\omega t + \frac{i\omega}{c}\sqrt{\epsilon\mu}\, z - \frac{2\pi\sigma}{c}\sqrt{\frac{\mu}{\epsilon}}\, z\right) \quad \text{—(12)}$$

$$\Delta z = \frac{c}{2\pi\sigma}\sqrt{\epsilon/\mu} \quad \text{—(13)}$$

$$\left.\begin{aligned} CB &= e_y n E e^{i(kz-\omega t)} \\ CB' &= e_y n' E' e^{i(k'z-\omega t)} \\ CB'' &= e_y n E'' e^{i(-kz-\omega t)} \end{aligned}\right\} \quad (2)$$

Clearly the reflected and transmitted waves must have the same frequency ω as the incident wave if boundary conditions at $z=0$ are to be satisfied for all t. The E-field must be continuous at the boundary.

$$E - E'' = E' \quad (3)$$

The H-field must ~~be~~ also be continuous and for nonmagnetic media ($\mu = \mu' = \mu_0$) so must be the B-field.

$$n(E + E'') = n'E' \quad (4)$$

eqn (3) and (4) can be solved simultaneously for the amplitudes E' and E'' in terms of the incident amplitude E.

UNIT - IV

plane wave in non-conducting medium:-

A non-conducting medium which has same properties in all directions is called an isotropic dielectric, maxwell equation are

$$\text{div } D = \nabla \cdot D = \rho$$

$$\text{div } B = \nabla \cdot B = 0$$

$$\text{Curl } E = \nabla \times E = -\partial B/\partial t$$

$$\text{curl } H = \nabla \times H = J + \partial D/\partial t$$

where $D = \epsilon E$; $B = \mu H$ $J = \sigma E$ and $\rho = 0$

Therefore, maxwell equations takes in the form,

$$\text{div } E = \nabla \cdot E = 0 \quad - (2a)$$

$$\text{div } H = \nabla \cdot H = 0 \quad - (2b)$$

$$\text{curl } E = -\mu \frac{\partial H}{\partial t} \quad - (2c)$$

$$\text{curl } H = \epsilon \, \partial E/\partial t \quad - (2d).$$

Taking curl of equation (2c),

$$\text{curl curl } E = -\mu \frac{\partial}{\partial t} (\text{curl } H)$$

The transverse parts of E and H satisfy the two curl equation (2c) and (2d) leading the transverse waves satisfies the equation.

from equations (2a) $\Rightarrow \frac{\partial E_l}{\partial x_\alpha} = 0$ — (9a)

from equations (2b) $\Rightarrow \frac{\partial H_l}{\partial x_\alpha} = 0$ — (9b)

from equation (2c) $\Rightarrow \left(\partial/\partial t + \frac{\sigma}{\varepsilon} \right) E_l = 0$ — (9c)

from equation (2d) $\Rightarrow \frac{\partial H_l}{\partial t} = 0$ — (9d)

from equation (9b) and (9d) shows the longitudinal magnetic field is static uniform field. equ (9a) and (9c) shows the longitudinal electric field in static uniform field.

consider the transverse field in conducting medium, the equ 5 and 6 can be written as,

$$E = E_0 \exp(ik r - i\omega t)$$

$$H = H_0 \exp(ik.r - i\omega t)$$

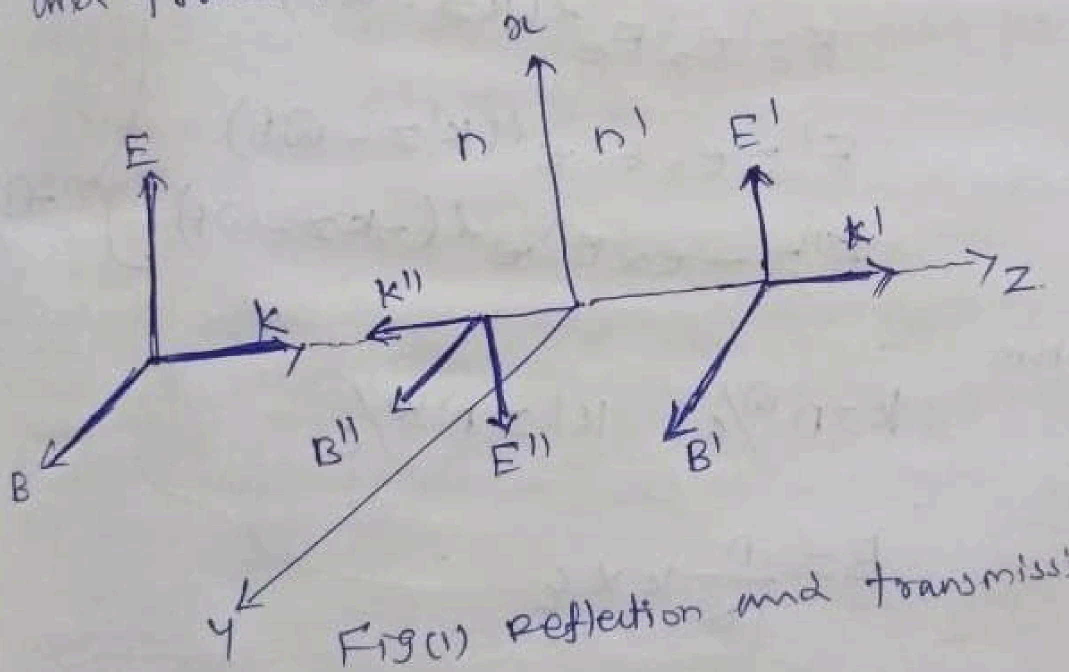
$$\text{and } \varphi = \varphi_0 \exp(ik.r - i\omega t) \quad \text{— (10)}$$

Where $k \rightarrow$ wave vector and E_0, H_0, φ_0 are complex variable.

Reflection and Refraction of electromagnetic waves at a plane interface between dielectrics.

Normal incidence:-

A plane wave normally incident on a plane dielectric interface. we will see that the boundary conditions are satisfied only if reflected and transmitted waves are present.



Fig(1) Reflection and transmission at normal incidence.

Fig(1) describes the incident wave (E, B) travelling in the z-direction, the reflected wave (E'', B'') travelling in the minus z-direction,

$$R = \frac{S''}{S} = |r|^2$$
$$= \left(\frac{n'-n}{n'+n}\right)^2,$$

$$T = \frac{S'}{S} = \frac{n'}{n}|t|^2$$
$$= \frac{4nn'}{(n'+n)^2} \quad \text{—— (8)}$$

with the Fresnel coefficients given by eqn (6)

R and T satisfy.

$$\boxed{R+T=1} \quad \text{—— (9)}$$

for any pair of nonconducting media, This is an expression of energy conservation at the interface.

— + —

(8) The radiation pressure P_{rad} applied by an electromagnetic wave on a perfectly absorbing surface turns out to be equal to the energy density of the wave.

$$\underbrace{P_{rad} = u}_{\text{perfect absorber}}$$

If the material is perfectly reflecting, such as a metal surface, and if the incidence is along the normal to the surface, then the pressure exerted is twice as much because the momentum direction reverses upon reflection.

$$\underbrace{P_{rad} = 2u}_{\text{Perfect reflector}}$$

$$[u] = J/m^3 = \frac{N \cdot m}{m^3} = N/m^2 = \text{unit pressure}$$

$$P = \langle P_{rad} \rangle = \begin{cases} I/c & \text{Perfect absorber} \\ 2I/c & \text{Perfect reflector} \end{cases}$$

— x —

Squaring and adding (13)

$$E_x^2 + E_y^2 = (E_1^{\circ})^2 + (E_2^{\circ})^2$$

$$\therefore \frac{E_x^2}{(E_1^{\circ})^2} + \frac{E_y^2}{(E_2^{\circ})^2} = 1$$

which is the equation of ellipse.

anomolous dispersion

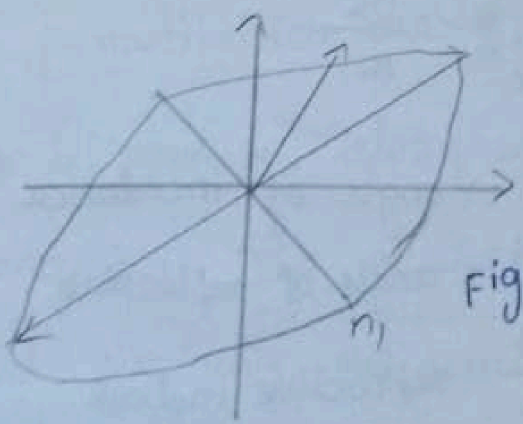


Fig 3 Elliptically polarised wave.

polarisation of Electromagnetic waves - Linearly and circular polarised :- (9)

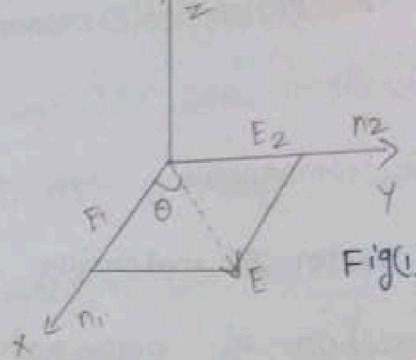
An electromagnetic wave in which the electric field vector E maintaing a fixed direction relative to direction of propagation is said to be linearly polarized.

The electric field (E) and magnetic field (H) vectors are $\perp^r$ to each other and also perpanticular to the direction of propagation of EM wave. There electromagnetic waves are called plane polarised.

The plane containing the direction of propagation and field vectors H is known as plane polarisation.

The plane containing the field vector E and direction of propagation is called plane of vibration

Any plane polarized wave can be considered to be sum of two plane polarised components in perpendicular direction, which is in phase



Fig(1) Linearly polarized wave

Figure shows the field vector E can be resolved into two components E_1 and E_2, which are perpenticular. These two independent components, which are in phase may be expressed as,

$$E_1 = n_1 E_1^0 \exp[ik.r - i\omega t] \quad -(1)$$

$$E_2 = n_2 E_2^0 \exp[ik.r - i\omega t] \quad -(2)$$

The general solution of plane polarised wave may be written as,

$$E(r,t) = (n_1 E_1^0 + n_2 E_2^0) \exp(ik.r - i\omega t) \quad -(3)$$

where E_1^0 and E_2^0 must have the same phase. so that the waves might be plane polarised. The polorisation vector E has magnitude.

$$E_0 = \sqrt{(E_1^0)^2 + (E_2^0)^2}$$

$$\theta = \tan^{-1}\left(\frac{E_2^0}{E_1^0}\right)$$

$$\therefore V = \frac{c}{\sqrt{K_m K_e}} \quad - (10)$$

(2)

Since
$K_m > 1$, $K_e < 1$

there by the speed of the EM wave is isotropic medium.

$$\text{put } n = \frac{c}{v} \Rightarrow v = \frac{c}{n} \quad - (11)$$

30/8/16.

compare equ (10) & (11)

$$n = \sqrt{K_m K_e}$$

when $K_m = 1$

$$n = \sqrt{K_e}$$

$$n^2 = K_e$$

Replace, $\mu \varepsilon = \frac{1}{v^2}$

$\therefore$ equ (5) & (6) can be written as,

$$\nabla^2 E - \frac{1}{v^2} \frac{\partial^2 E}{\partial t^2} = 0 \quad - (12)$$

$$\nabla^2 H - \frac{1}{v^2} \frac{\partial^2 H}{\partial t^2} = 0 \quad - (13)$$

The plane wave solution of equation (12) and (13) is well known from, These are the vector equation in identical form of E and H satisfied the scalar equation, (3)

$$\nabla^2 u - \mu\varepsilon \frac{\partial^2 u}{\partial t^2} = 0 \quad \text{--- (7)} \qquad \therefore u \rightarrow \text{scalar form.}$$

put $\mu\varepsilon = \frac{1}{v^2}$

equation (7) becomes

$$\nabla^2 u - \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2} = 0 \quad \text{--- (8)}$$

where

$v \rightarrow$ speed of the wave. This means that the field vector E and H are propagated in isotropic dielectric as waves with speed 'v'.

$$\therefore v = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$v = \frac{1}{\sqrt{K_m \mu_0 K_e \varepsilon_0}} \quad \text{--- (9)}$$

where,

$K_m \rightarrow$ relative permeability of medium.

$K_e \rightarrow$ relative permittivity of medium.

as $\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ is called speed of electromagnetic waves in free space.

Substitute this in equ (2d)

$$\text{curl curl } E = -\mu \, \partial/\partial t \left(\epsilon \frac{\partial E}{\partial t}\right)$$

$$\text{curl curl } E \; -\mu\epsilon \frac{\partial^2 E}{\partial t^2} \quad - (3)$$

from the equation (2a) and (2b)

Taking curl of equ (2d),

$$\text{curl curl } H = \epsilon \, \partial/\partial t \, (\text{curl } E)$$

Substitute this in equ (2c)

$$\text{curl}(\text{curl } H) = \epsilon \, \partial/\partial t \left(-\mu \frac{\partial H}{\partial t}\right)$$

$$\text{curl}(\text{curl } H) = -\mu\epsilon \frac{\partial^2 H}{\partial t^2} \quad - (4)$$

using vector identity.

$$\text{curl curl } A = \text{grad} . \text{div } A - \nabla^2 A$$

from the equ (2a) and (2b)

$$\text{div } E = 0 \text{ and } \text{div } H = 0.$$

$\therefore$ equ (3) and (4) becomes

$$\nabla^2 E - \mu\epsilon \frac{\partial^2 E}{\partial t^2} = 0 \quad - (5)$$

$$\nabla^2 H - \mu\epsilon \frac{\partial^2 H}{\partial t^2} = 0 \quad - (6)$$

$$E(r,t) = E_0 \exp(ik.r - i\omega t) \quad -(14)$$

$$H(r,t) = H_0 \exp(ik.r - i\omega t) \quad -(15)$$

where E_0 and H_0 are

$\therefore$ complex amplituder. equation (14) and (15) represent the plane waves in non conducting medium.

6. plane electromagnetic waves in conducting medium:-

The plane waves in conducting medium can be derived from maxwell's equations

The maxwell equations are,

$$\text{div } D = \nabla . D = \rho$$

$$\text{div } B = \nabla . B = 0$$

$$\text{curl } E = -\partial B/\partial t$$

$$\text{curl } H = J + \partial D/\partial t \quad -(1)$$

Let us assume that the medium is linear and isotropic and is characterised by permitivity ε and permeability μ and conductivity σ.

but not any charge (or) any current other than determined by ohm's law,

$$D = \varepsilon E \quad , B = \mu H \text{ and } J = \sigma E \text{ and } \rho = 0$$

equ ① becomes.

$$\text{div } E = 0 \rightarrow (2a)$$

$$\text{div } H = 0 \rightarrow (2b)$$

$$\text{curl } E = -\mu \frac{\partial H}{\partial t} \rightarrow (2c)$$

$$\text{curl } H = \sigma E + \varepsilon \frac{\partial E}{\partial t} \quad -(2d)$$

Taking curl of equation (2c)

$$\text{curl curl } E = -\mu \frac{\partial}{\partial t} (\text{curl } H)$$

Substitute equ (2d)

$$\text{curl curl } E = -\mu \, \partial/\partial t \left(\sigma E + \varepsilon \frac{\partial E}{\partial t} \right)$$

$$\text{curl } (\text{curl } E) = -\sigma\mu \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \quad -③$$

Similarly, taking curl of (2d), and substitute (2c) we get

$$\text{curl curl } H = -\sigma\mu \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} \quad -④$$

using vector identify

$$\text{curl.curl } A = \text{grad div } A - \nabla^2 A$$

keeping equations (2a) and (2b) in equation (3) and ④

we get,

(7)

$$\nabla^2 E - \sigma\mu \frac{\partial E}{\partial t} - \epsilon\mu \frac{\partial^2 E}{\partial t^2} = 0 \quad -(5)$$

$$\nabla^2 H - \sigma\mu \frac{\partial H}{\partial t} - \epsilon\mu \frac{\partial^2 H}{\partial t^2} = 0 \quad -(6)$$

These equations represents wave equation with electromagnetic field E and H is, homogeneous istropic conducting medium of conducting σ.

It is apparent these equation and vector equation of identical form, which satisfies the scalar wave equation

$$\nabla^2 \psi - \sigma\mu \frac{\partial \psi}{\partial t} - \epsilon\mu \frac{\partial^2 \psi}{\partial t^2} = 0 \quad -(7)$$

In an isotropic dielectric, we have seen that the time varying fields are transverse If the conductivity 'σ' is not zero. Let us assume the field vary only are spatial variable x_α.

$\therefore$ The field equations in electric and magnetic vectors can be written as,

$$E(x_\alpha, t) = E_t(x_\alpha, t) + E_l(x_\alpha, t)$$

$$H(x_\alpha, t) = H_t(x_\alpha, t) + H_l(x_\alpha, t) \quad -(8)$$

Thus it fixed point in space of equ (6), The electric field vector but sweeps around in circle at a frequency 'ω'. For upper sign of $(n_1 \pm i n_2)$ the rotation is an clockwise, the observer is facing into on coming wave. This is called left handed circularly polarised (or) positive helicity.

16 Thus it is fixed point in space of equ(6). The electric field vector but sweeps around in the circle at frequency 'ω', for lower sign of $(n_1 \pm i n_2)$. The rotation is clockwise, the observer in facing into towards passive wave This called right handed circularly polarized (or) negative helicity.

Negative helicy. If the amplitude E_1^0 and E_2^0 are not equal in magnitude, Then the equ(6) may be written as,

$$\left.\begin{aligned} E_x(r,t) &= E_1^0 \cos(kz-\omega t) \\ E_y(r,t) &= \pm E_2^0 \cos(kz-\omega t) \end{aligned}\right\} - (7)$$

If the waves E_1° and E_2° have different phase, The wave might be called elliptically polarised (or) circularly polarised. (11)

If E_1° and E_2° have the same real magnitude E_0 (say) differ in phase 90°, equ(2) may be written as,

$$E(r,t) = E^\circ (n_1 \pm i n_2) \exp[ik \cdot r - i\omega t] \quad - (12)$$

If the wave is propagating in z direction, which E and E_2 are along x and y direction, the components of electric field of equ (4) is,

$$E_x(r,t) = E_0 \cos(kz - \omega t)$$
$$E_y(r,t) = E_0 \sin(kz - \omega t) \quad - (5)$$

squaring and adding,

$$E_x^2 + E_y^2 = E_0^2 (1)$$

$$\frac{E_x^2}{E_0^2} + \frac{E_y^2}{E_0^2} = 1 \quad - (6)$$

which is known as equation of circle.

Fig (2a) : circularly polarised anti clockwise.

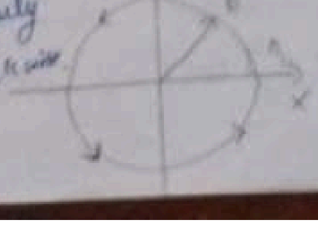
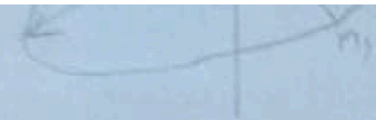


Fig (2b) : circularly polarised clockwise.



wave.

Reflection and refraction of Electromagnetic waves as the interface of non conducting media:-

The reflection and refraction of light at a plane interface between two media of different dielectric properties are similar phenomenon.

The various types of the components

1) Kinematic properties

2) Dynamic properties.

Kinematic properties:-

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$$E'' = \frac{n'-n}{n'+n} E, \qquad E' = \frac{2n}{n'+n} E \quad \text{—— (5)}$$

The Fresnel coefficients for normal incidence reflection and transmission are defined as,

$$r = \frac{E''}{E} = \frac{n'-n}{n'+n}, \quad t = \frac{E'}{E} = \frac{2n}{n'+n} \quad \text{—— (6)}$$

For $n' > n$, there is a phase reversion for the reflected wave.

it is usually measureable is the reflected and transmitted average energy fluxes per unit area (a.k.a. the intensity of Em wave) given by the magnitude of the poynting vector.

$$S = \tfrac{1}{2}\left| E \times H^* \right| = \tfrac{1}{2} n c \epsilon_0 |E|^2 \quad \text{—— (7)}$$

we define the reflectance R and the transmittance T for normal incidence by the ratios of the intensites.

Plane waves in a conducting medium

If the medium in which our wave is propagating is a conductor, the electric field will generate a conduction current.

$$J_F = \sigma E \quad \text{--- (1)}$$

and this will affect the propagation of the wave. with the inclusion of the conduction current. From the maxwell's eqn.

$$\nabla \cdot E = 0 \quad \text{--- (2)}$$

$$\frac{1}{\mu} \nabla \times B = \frac{\varepsilon}{c}\frac{\partial E}{\partial t} + \frac{4\pi\sigma}{c} E \quad \text{--- (3)}$$

$$\nabla \cdot B = 0 \quad \text{--- (4)}$$

$$\nabla \times B = -\frac{1}{c}\frac{\partial B}{\partial t} \quad \text{--- (5)}$$

where we made the usual assumption that the medium carries no excess of free charge, so,

$\rho_F = 0$, For an osillating electric field with the time dependence $e^{-i\omega t}$ we have

$$\frac{\partial E}{\partial t} = -i\omega E \quad \text{--- (6)}$$

(or) $$E = \frac{i}{\omega}\frac{\partial E}{\partial t} \quad \text{--- (7)}$$

eqn (3) can be replaced by

(i) Law of reflection :-

The angle of reflection is equal to the angle of incidance.

$$\theta r = \theta i$$

(ii) Snell's Law :-

$$\frac{\sin \theta i}{\sin \theta r} = \frac{n_2}{n_1}$$

where $\theta i \rightarrow$ angle of incidance

$\theta r \rightarrow$ angle of reflection

$n_1, n_2 \rightarrow$ refractive indices.

(iii) Law of frequency :-

The incident, reflected and refracted waves all have the some frequency.

2) Dynamic properties :-

Let us consider a plane interface at $z=0$ separating two homogeneous charge free and non conducting isotropic media

Electromagnetic wave Propagation b/w conducting plates

* A waveguide may refer to any structure that conveys electromagnetic waves between its endpoints.

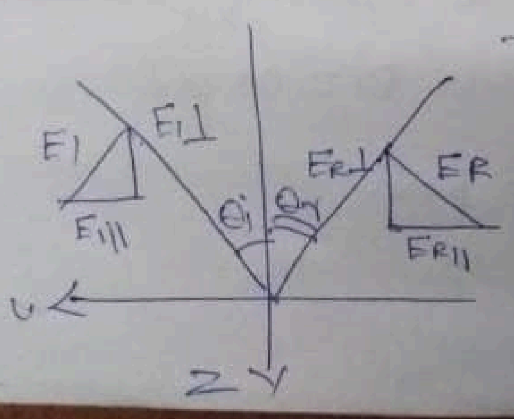
* Transverse Electric (TE) modes have $E \perp k_g$ (Propagation wavevector)

* Transverse magnetic (TM) modes have $B \perp k_g$

* Transverse Electric - magnetic modes (TEM) have $E, B \perp k_g$.

* A cutoff frequency exists, below which no radiation propagates.

Electromagnetic wave reflection by perfect conductor:

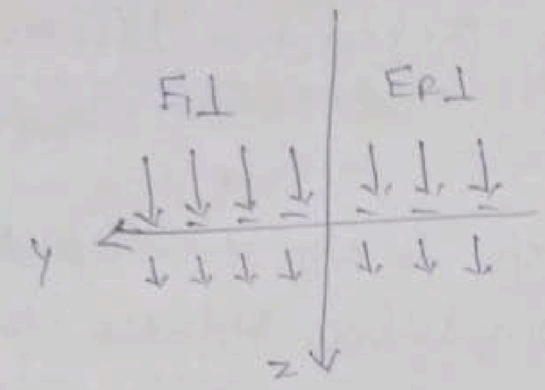


- $E_\perp$ can be finite just outside conducting surface
- $E_\parallel$ vanishes just outside and inside conducting surface

(14)

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}\left(\vec{r}', t - |\vec{r} - \vec{r}'|\right)}{|\vec{r} - \vec{r}'|} d^3r'.$$

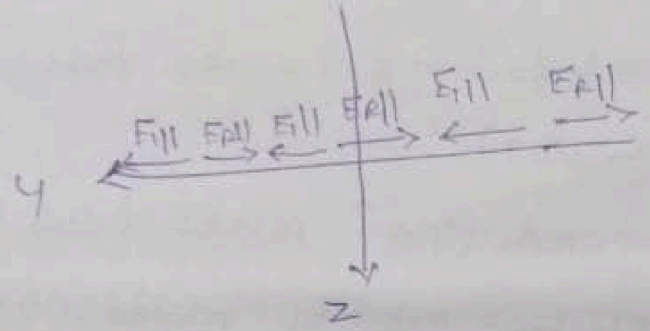
— x —



$D_{\perp 1} = D_{\perp 2}$

$D_{\perp 1} = \varepsilon_0 E_{\perp 1}$

$D_{\perp 2} = \varepsilon_0 \varepsilon E_{\perp 2}$



$E_{\parallel 1} = E_{\parallel 2}$

$E_{01} + E_{0R} = 0$

$E_{0T} = 0$

Boundary conditions $B_{\perp 1} = B_{\perp 2}$

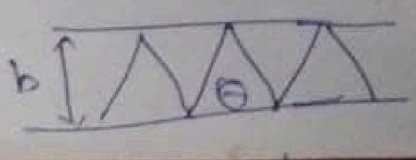
$E_{\parallel 1} = E_{\parallel 2}$ (1, 2 inside, outside here)

$E_{\parallel}$ must vanish just outside conducting surface since $E = 0$ inside

$E_\perp$ may be finite just outside since induced surface charges allow $E = 0$ inside (TM modes only)

$B_\perp = 0$ at surface since $B_1 = 0$.

Two parallel plates, TE mode



Dipole

* Electric dipole' deals with the separation of the positive and negative charges found in any electromagnetic system.

A simple example of this system is a pair of electric charges of equal magnitude but opposite sign separated by some typically small distance. (A permanent electric dipole is called an electret).

* A magnetic dipole is a closed circulation of an electric current system. A simple example is a single loop of wire with constant current through it. A bar magnet is an example of a magnet with a permanent magnetic dipole moment

— x —

The incident wave

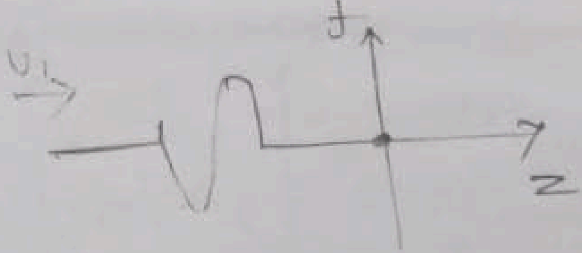
$$\vec{f}_I(z,t) = \bar{A}_I e^{i(k_1 z - \omega t)} \quad (z<0)$$

reflected wave

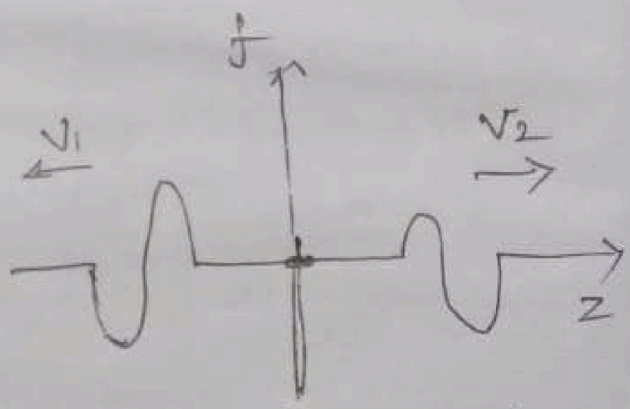
$$\vec{f}_R(z,t) = \bar{A}_R e^{i(k_1 z - \omega t)}$$

transmitted wave $\vec{f}_T(z,t) = \bar{A}_T e^{i(k_2 z - \omega t)}$ $(z>0)$

$$\frac{\lambda_1}{\lambda_2} = \frac{k_2}{k_1} = \frac{v_1}{v_2}$$



(a) Incident pulse.



(b) Reflected and transmitted pulse

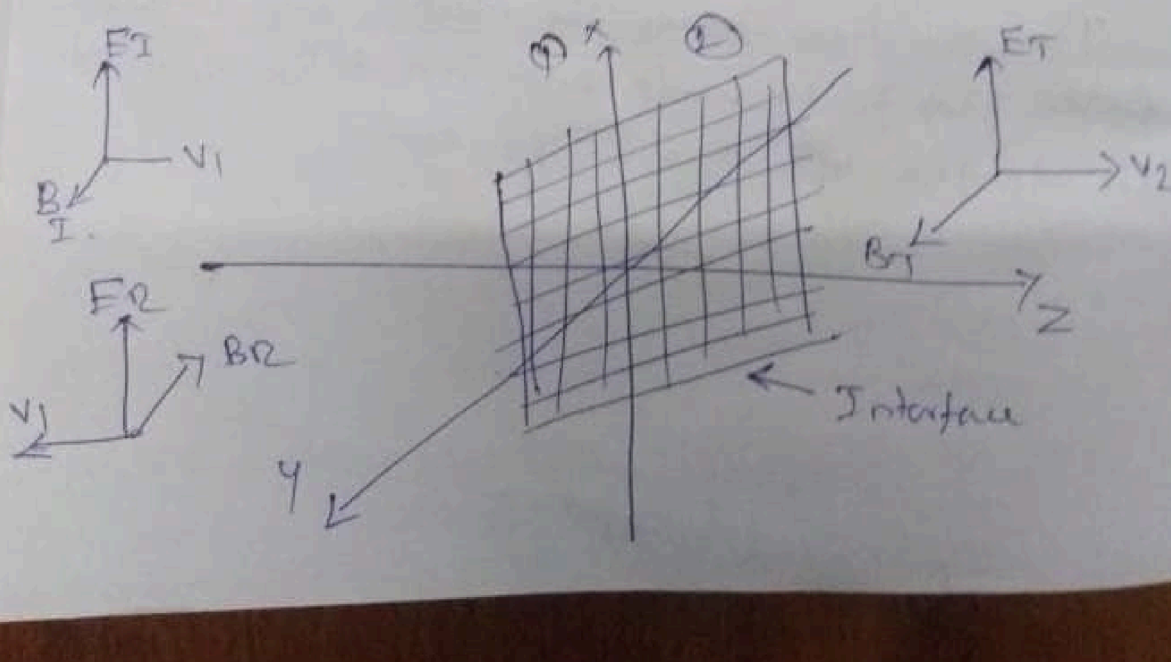
Reflection and Transmission at Normal Incidence.

The xy plane forms the boundary b/w two linear media, A plane wave of frequency ω, traveling in the z direction and polarized in the x direction, approaches the interface from the left,

$$\left.\begin{aligned} \vec{E}_I(z,t) &= \vec{E}_{0I}\, e^{i(k_1 z - \omega t)}\, \hat{x}, \\ \vec{B}_I(z,t) &= \frac{1}{v_1}\vec{E}_{0I}\, e^{i(k_1 z - \omega t)}\, \hat{y} \end{aligned}\right\} \text{①}$$

It gives rise to a reflected wave

$$\left.\begin{aligned} \vec{E}_R(z,t) &= \vec{E}_{0R}\, e^{i(-k_1 z - \omega t)}\, \hat{x} \\ \vec{B}_R(z,t) &= -\frac{1}{v_1}\vec{E}_{0R}\, e^{i(-k_1 z - \omega t)}\, \hat{y} \end{aligned}\right\} \text{②}$$



Radiation pressure and momentum.

The direction of the force for a very specific case, consider a plane electromagnetic wave incident on a metal in which electron motion, as part of a current, is damped by the resistance of the metal, so that the average electron motion is in phase with the force causing it. This is comparable to an object moving against friction and stopping as soon as the force pushing it stops. fig(1).

When the electric field is in the direction of the positive Y-axis, electrons move in the negative Y-direction, with the magnetic field in the direction of the positive Z-axis. By applying the right hand rule, and accounting for the negative charge of the electron, we can see that the force on the electron from the magnetic field is in the direction of the positive X-axis - which is the direction of wave propagation. When the $\vec{E}$ field reverse, the $\vec{B}$ field does too, and the force is again in the same direction.

Maxwell's eqn together with the Lorentz force equation imply the existence of radiation pressure much more generally than this specific example

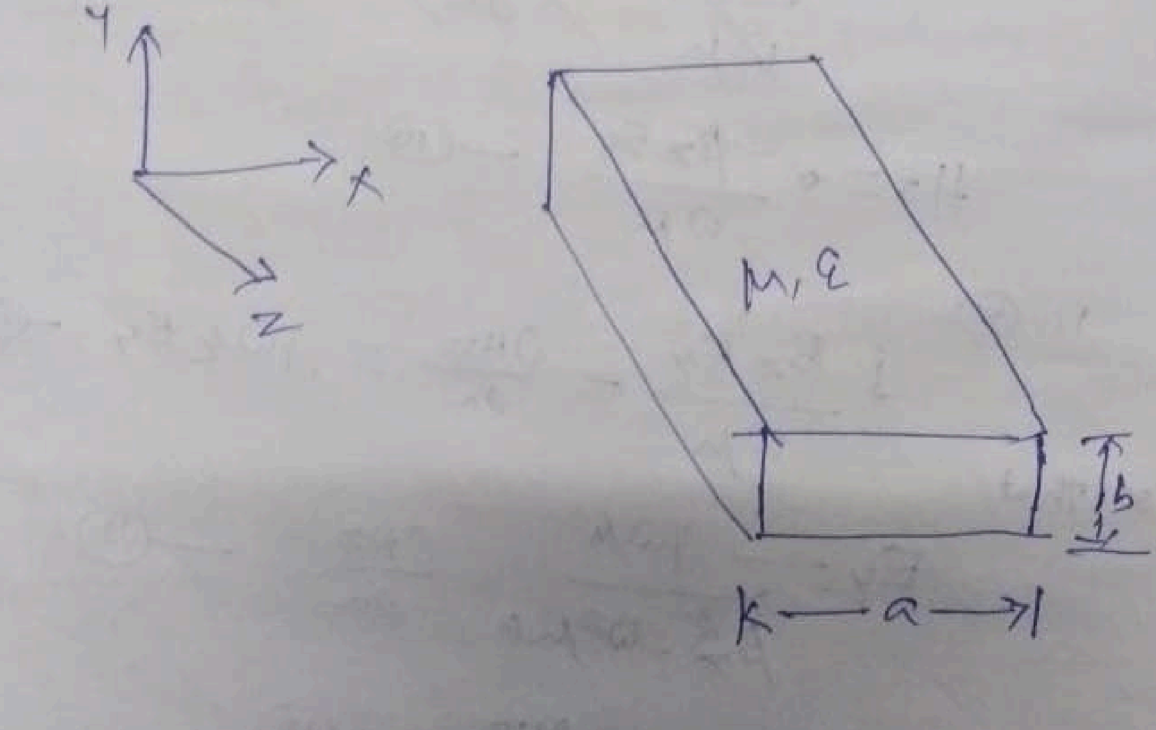
$$H_y = \frac{j\beta_z}{\beta_z^2 - \omega^2\mu\varepsilon} \frac{\partial H_z}{\partial y} \quad \text{—— (14)}$$

$$E_z = 0 \quad \text{—— (15)}$$

combining solutions for E_x and E_y into (8) gives

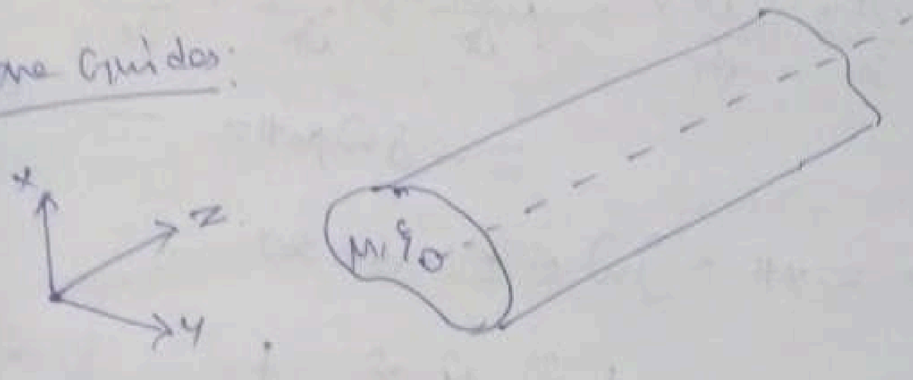
$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} = \left[\beta_z^2 - \omega^2\mu\varepsilon\right] H_z \quad \text{—— (16)}$$

Rectangular waveguides:



wave Guides of Rectangular cross-section

wave Guides:



Maxwell's eqn:

$$\nabla^2 E + \omega^2 \mu \epsilon E = 0 \quad \text{— (1)}$$

$$\nabla^2 H + \omega^2 \mu \epsilon H = 0 \quad \text{— (2)}$$

For a waveguide with arbitary cross section as shown in the above fig. we assume a plane wave solution and as a first trail,

We set $E_z = 0$. This defines the TE mode.

From $\nabla \times E = -\mu \frac{\partial H}{\partial t}$ we have,

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\mu \frac{\partial H_x}{\partial t} \Rightarrow + j \beta_z E_y$$
$$= -j\omega\mu H_x \quad \text{— (3)}$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\mu \frac{\partial H_y}{\partial t} \Rightarrow -j\beta_z E_x = -j\omega\mu H_y \quad \text{— (4)}$$

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \beta_z^2 = \omega^2\mu\epsilon \quad \text{—} (24)$$

and

$$\beta_z = \omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 \quad \text{—} (25)$$

The guidance condition is

$$\omega^2\mu\epsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \quad \text{—} (26)$$

(a)

$f > f_c$ where f_c is the cutoff frequency of the TE_{mn} mode given by the relation.

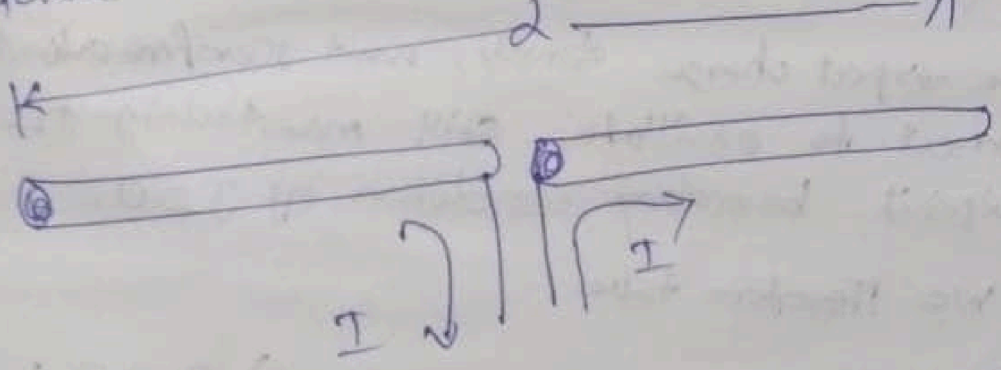
$$f_c = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \quad \text{—} (27)$$

The TE_{mn} mode will not propagate unless f is greater than f_c, obviously, different modes will have diff cut of cutoff frequencies

$$\boxed{E_z = E_0 e^{-j\beta_z z} \sin \beta_x x \sin \beta_y y} \quad (30)$$

Center fed linear Antenna :-

Let us consider the radiation arising from the thin linear, center-fed antenna illustrated in fig. We choose the z axis to lie along the antenna.



The antenna of length d is split by a small gap at its ~~midd~~ midpoint where each half is supplied by current $\pm I_0 e^{-i\omega t}$.

To deduce the magnitude of the current on any point of the antenna. We neglect radiation damping. The current must be symmetric about the gap in the middle. And further, it must vanish at the ends.

Dielectric Wave Guide

The transverse behaviour of the fields is governed by two eqns like one one for inside the cylinder and one for outside.

Inside
$$\left[\nabla_t^2 + \left(\mu_1 \epsilon_1 \frac{\omega^2}{c^2} - k^2\right)\right]\left\{\begin{matrix} E \\ B \end{matrix}\right\} = 0 \quad -(1)$$

outside
$$\left[\nabla_t^2 + \left(\mu_0 \epsilon_0 \frac{\omega^2}{c^2} - k^2\right)\right]\left\{\begin{matrix} E \\ B \end{matrix}\right\} = 0$$

Both dielectric (μ_1, ϵ_1) and surrounding medium (μ_0, ϵ_0) are assumed to be uniform and is isotropic in their properties.

The axial propagation constant k must be the same inside and outside the cylinder in order to satisfy boundary conditions at all points on the surface at all times.

inside the dielectric cylinder the transverse Laplacian of the fields must be negative so that the constant

$$\nu^2 = \mu_1 \epsilon_1 \frac{\omega^2}{c^2} - k^2$$ is positive

(13)

If the antenna were short, we would expect the entire right side to uniformly charged ~~to one~~ to one polarity, while another side would would be uniformly charged with the opposite polarity. For the one-dimensional problem, the continuity eqn then states $\partial J/\partial z = -\partial \rho/\partial t$ = constant.

Thus J would be of the form (const) $\times$

$$(\tfrac{1}{2}d - |z|)\, \delta(x)\, \delta(y).$$

For a linear antenna we expect charge density, and therefore also the current, to oscillate, still maintaining the endpoint boundary condition of $J = 0$.

We therefore take

$$\vec{J}(r,t) = \hat{k}\, J \sin(\tfrac{1}{2}kd - k|z|)\, \delta(x)\, \delta(y)\, e^{-i\omega t}$$

$$\text{for } |z| \leq \tfrac{1}{2}d$$

The supply current will evidently be $I_0 e^{-i\omega t} = J e^{-i\omega t} \sin \tfrac{1}{2}kd$. The vector potential due to an oscillating current is in general given by, The other components of E and B can be found from.

Inside
$$\left.\begin{array}{ll} B_\rho = \dfrac{ik}{\gamma^2}\dfrac{\partial B_z}{\partial \rho}, & B_\phi = \dfrac{i\epsilon_1\omega}{\gamma^2 c}\dfrac{\partial E_z}{\partial \rho} \\ E_\phi = -\dfrac{\omega}{ck}B_\rho & E_\rho = \dfrac{ck}{\epsilon_1\omega}B_\phi \end{array}\right\}$$

$$\left.\begin{array}{l} B_z = J_0(\gamma\rho) \\ B_\rho = -\dfrac{ik}{\gamma}J_1(\gamma\rho) \\ E_\phi = \dfrac{i\omega}{c\gamma}J_1(\gamma\rho) \end{array}\right\} \rho \le a$$

and
$$B_z = AK_0(\beta\rho)$$
$$B_\rho = \frac{ikA}{\beta}K_1(\beta\rho)$$
$$E_\phi = -\frac{i\omega A}{c\beta}K_1(\beta\rho)$$

— + —

(16) **Electric dipole fields.**

Once the time-dependent potentials are electric field and magnetic field can be calculated in the usual way. Namely,

$$E(x,t) = -\nabla\phi(x,t) - \frac{\partial A(x,t)}{\partial t}$$

$$B(x,t) = \nabla \times A(x,t)$$

(or)

In a source-free region of space, the relationship b/w magnetic field and the electric field can be used to obtain

$$H(x,t) = \frac{1}{\mu_0}\nabla \times A(x,t)$$

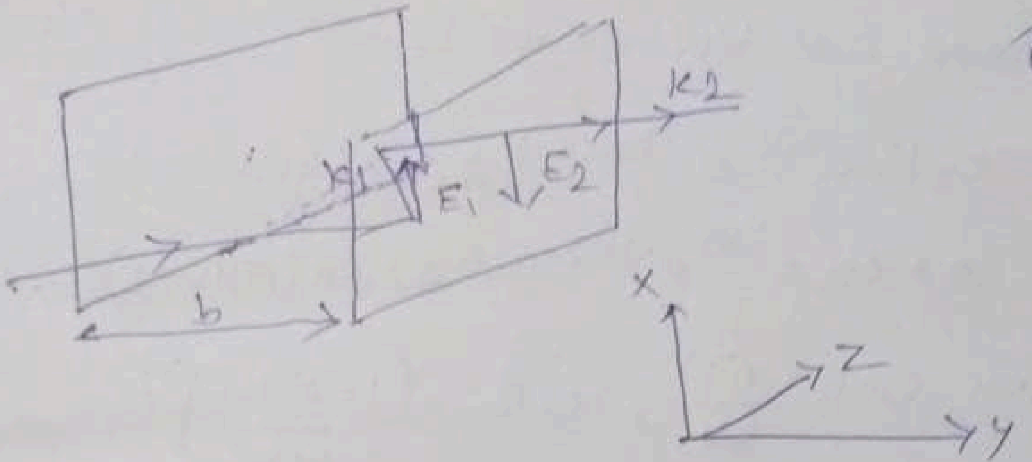
$$E(x,t) = \frac{iZ_0}{k}\nabla \times H(x,t)$$

where $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$ is the impedance of freespace

The electric and magnetic fields that correspond to the potentials above are.

$$H_{\text{Electric dipole}}(x,t) = \frac{ck^2}{4\pi}(n \times p)\, e\frac{ikr-i\omega t}{r}$$

$E_{\text{Electric dipole}}(x,t) = Z_0(H_{\text{Electric dipole}}$ which is consistent with spherical radiation waves.



$$E = E_1 + E_2$$

$$= e_x E_0 e^{i\omega t}\left(e^{i(-ky\sin\theta + kz\cos\theta)} - e^{i(ky\sin\theta + kz\cos\theta)}\right)$$

$$= e_x E_0 e^{i\omega t} e^{-ikz\cos\theta} 2i\sin(ky\sin\theta)$$

Boundary condition $E_{||1} = E_{||2} = 0$.

means that $E = E_{||}$ vanishes at $y=0, y=b$.

$E_{||}(y = 0, b)$ if $ky\sin\theta = n\pi$

$n = 1, 2, 3 \ldots$

$$k = \frac{n\pi}{b\sin\theta} \qquad \sin\theta = \frac{n\pi}{kb}$$

$E_1 = e_x E_0 e^{j(\omega t - k_1 \cdot r)}$

$E_1 = -e_y k\sin\theta + e_z k\cos\theta$

$k_1 \cdot r = -ky\sin\theta + kz\cos\theta$

$E_2 = -e_x E_0 e^{i(\omega t - k_2 \cdot r)}$

$k_2 = +e_y k\sin\theta + e_z k\cos\theta$.

$k_2 \cdot r = +ky\sin\theta + kz\cos\theta$

(17)

$$A_{\text{magnetic dipole}}(x,t) = \frac{-ik\mu_0}{4\pi} \frac{e^{ikr - i\omega t}}{r} m \times n.$$

Notice that A magnetic dipole has a similar form to H Electric dipole.

That means the magnetic field from a magnetic dipole behaves similarly to the electric field from an electric dipole. Likewise, the electric field from a magnetic dipole behaves like the magnetic field from an electric dipole. Taking the transformation.

$$E_{\text{Electric dipole}} \rightarrow Z_0 H_{\text{magnetic dipole}}$$

$$H_{\text{Electric dipole}} \rightarrow -\frac{1}{Z_0} E_{\text{magnetic dipole}}$$

$$p \rightarrow m/c.$$

<u>magnetic dipole fields:</u>

$$E_{\text{magnetic dipole}}(x,t) = \frac{-k^2 Z_0}{4\pi} n \times m \frac{e^{ikr - i\omega t}}{r}$$

$$H_{\text{magnetic dipole}}(x,t) = \frac{-1}{Z_0}\left(E_{\text{magnetic dipole}} \times n\right)$$

TM modes

The (transverse magnetic) modes for a general waveguide are obtained by assuming $H_z = 0$, By duality with the TE mode, we have,

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} = \left[\beta_z^2 - \omega^2 \mu \epsilon\right] E_z \quad \text{--- (28)}$$

with general solution

$$E_z = e^{-j\beta_z z}\left[A e^{-j\beta_x x} + B e^{+j\beta_x x}\right]\left[C e^{-j\beta_y y} + D e^{+j\beta_y y}\right] \quad \text{--- (29)}$$

The boundary conditions are

At $x = 0$, $E_z = 0$ which leads to $A = -B$

At $y = 0$, $E_z = 0$ which leads to $C = -D$.

At $x = a$, $E_z = 0$ which leads to $\beta_x = \frac{m\pi}{a}$.

At $y = b$, $E_z = 0$ which leads to $\beta_y = \frac{n\pi}{b}$.

So that the generating eqn for the TM_{mn} modes is

At $y=0$, $E_x=0$ which leads to $C=D$

At $x=0$, $E_y=0$ which leads to $A=B$

$$H_z = H_0 e^{-j\beta_z z} \cos\beta_x x \cos\beta_y y \quad \text{--- (20)}$$

$$E_y = \frac{j\beta_x \omega\mu}{\beta_z^2 - \omega^2\mu\epsilon} H_0 e^{-j\beta_z z} \sin\beta_x x \cos\beta_y y. \quad \text{--- (21)}$$

$$E_x = \frac{-j\beta_y \omega\mu}{\beta_z^2 - \omega^2\mu\epsilon} H_0 e^{-j\beta_z z} \cos\beta_x x \sin\beta_y y. \quad \text{--- (22)}$$

At $x=a$, $E_y=0$ this leads to $\beta_x = \frac{m\pi}{a}$

At $y=b$, $E_x=0$ this leads to $\beta_y = \frac{n\pi}{b}$

The dispersion relation of is obtained by placing (20) in (17)

$$\beta_x^2 + \beta_y^2 + \beta_z^2 = \omega^2\mu\epsilon \quad \text{--- (23)}$$

From 4 we have

$$H_y = \frac{\beta_z E_x}{\omega \mu} \quad \text{—— (7)}$$

in 4

$$\frac{\partial H_z}{\partial y} + j\beta_z^2 \frac{E_x}{\omega \mu} = j\omega \epsilon E_y \quad \text{—— (8)}$$

solving for E_x

$$E_x = \frac{j\omega \mu}{\beta_z^2 - \omega^2 \mu \epsilon} \frac{\partial H_z}{\partial y} \quad \text{—— (9)}$$

From 1

$$j \frac{\cancel{\beta_z^2 E_y}}{\cancel{\omega \mu}} - \cancel{\frac{\partial H_z}{\partial x}} \cancel{=} j\omega \cancel{\epsilon E_y} -$$

$$H_x = \frac{-\beta_z E_y}{\omega \mu} \quad \text{—— (10)}$$

in 5

$$j \frac{\beta_z^2 E_y}{\omega \mu} - \frac{\partial H_z}{\partial x} = j\omega \epsilon E_y \quad \text{—— (11)}$$

so that

$$E_y = \frac{-j\omega \mu}{\beta_z^2 - \omega^2 \mu \epsilon} \frac{\partial H_z}{\partial x} \quad \text{—— (12)}$$

$$H_x = \frac{j\beta_z}{\beta_z^2 - \omega^2 \mu \epsilon} \frac{\partial H_z}{\partial x} \quad \text{—— (13)}$$

If the cross section of the waveguide is a rectangle, we have a rectangular waveguide and the ~~form of~~ boundary conditions are such that tangential electric field is zero on all the PEC walls. 8

<u>TE modes</u> (Tangential Electric)'

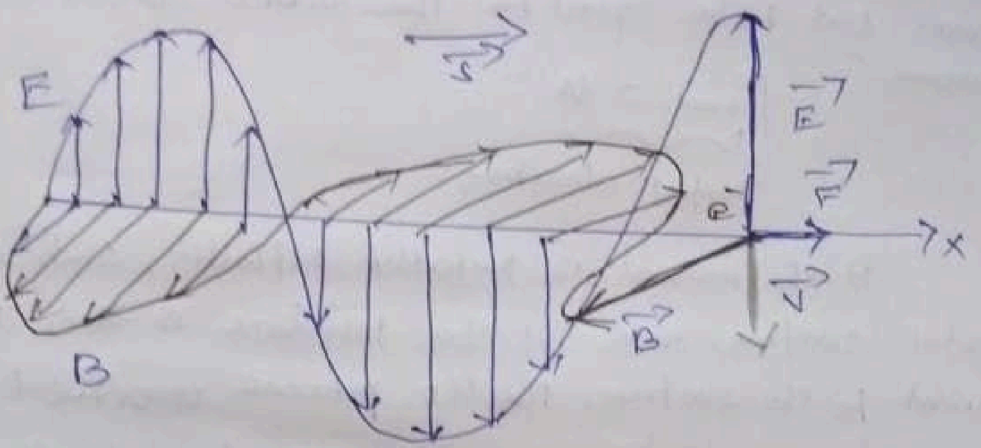
The general solution for TE modes with $E_z = 0$ is obtained from 16.

$$H_z = e^{-j\beta_z z}\left[A e^{-j\beta_x x} + B e^{+j\beta_x x}\right]\left[C e^{-j\beta_y y} + D e^{+j\beta_y y}\right] \quad \text{— 17}$$

$$E_y = \frac{\beta_x \omega \mu}{\beta_z^2 - \omega^2 \mu \epsilon} e^{-j\beta_z z}\left[-A e^{-j\beta_x x} + B e^{+j\beta_x x}\right]\left[C e^{-j\beta_y y} + D e^{+j\beta_y y}\right] \quad \text{— 18}$$

$$E_x = \frac{-\beta_y \omega \mu}{\beta_z^2 - \omega^2 \mu \epsilon} e^{-j\beta_z z}\left[A e^{-j\beta_x x} + B e^{+j\beta_x x}\right]\left[-C e^{-j\beta_y y} + D e^{+j\beta_y y}\right] \quad \text{— 19}$$

however



Fig(1)

Electric and magnetic fields of an electromagnetic wave can combine to produce a force in the direction of propagation, as illustrated for the special case of electrons whose motion is highly damped by the resistance of a metal.

Maxwell predicted that an electromagnetic waves carries momentum. An object absorbing an electromagnetic wave would experience a force in the direction of propagation of the wave. The force corresponds to radiation pressure exerted on the object by the wave. The force would be twice as great if the radiation were reflected rather than absorbed.

For monochromatic plane waves the frequency and wave number are related by $\omega = kv$ (25)

The amplitude of B is $\frac{1}{v}$ times the amplitude of E and the intensity is

$$I = \frac{1}{2} \epsilon v E_0^2$$

$$\left.\begin{aligned}
&(i)\ \epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp} \\
&(ii)\ B_1^{\perp} = B_2^{\perp} \\
&(iii)\ E_1^{\parallel} = E_2^{\parallel} \\
&(iv)\ \frac{1}{\mu_1} B_1^{\parallel} = \frac{1}{\mu_2} B_2^{\parallel}
\end{aligned}\right\}$$

These equations relate the electric and magnetic fields just to the left and just to the right of the interface b/w two linear media.

Allowed field b/w guides is

$$E = e_x E_0 e^{i\omega t} e^{-ikz\cos\theta} 2i \sin(ky\sin\theta)$$

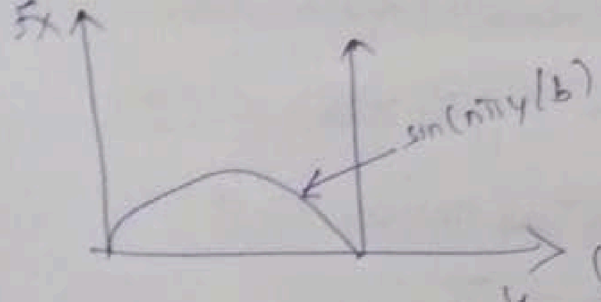
$$= e_x E_0 e^{i\omega t} e^{-ikz\cos\theta} 2i \sin(n\pi y/b)$$

since $\sin\theta = \frac{n\pi}{kb}$, $\cos\theta = \left(1 - \frac{n^2\pi^2}{k^2b^2}\right)^{1/2}$

The wave number for the guided field is

$$k_g = k\cos\theta = \left(k^2 - \frac{n^2\pi^2}{b^2}\right)^{1/2}$$

$n = 1, 2, 3 \ldots$



(TE₁)

Fields

$E_1 = e_x E_0 e^{i(\omega t - k_1 \cdot r)}$

$k_1 = -e_y k\sin\theta + e_z k\cos\theta$

$k_1 \cdot r = -ky\sin\theta + kz\cos\theta$

$E_2 = -e_x E_0 e^{i(\omega t - k_2 \cdot r)}$

$k_2 = +e_y k\sin\theta + e_z k\cos\theta$

$k_2 \cdot r = +ky\sin\theta + kz\cos\theta$.

—x—

The component of E tangential to the interface is continuous across the interface.

There is no reason to suspect that E becomes infinite at the boundary and so the potential is continuous across the interface as a consequence of its definition.

The Potential of a localized charge distribution.

The electric field is $E = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{1}{r^2}\right)\hat{r}$ and

$$dl = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

So,

$$E \cdot dl = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}dr.$$

setting the reference point at infinity, the potential of a point charge q at the origin is

$$V(r) = -\int_0^\infty E\,dl = -\frac{1}{4\pi\epsilon_0}\int_\infty^r \frac{q}{r'^2}\,dr'$$

$$\left[\because \nabla^2 V = -\frac{\rho}{\epsilon_0}\right.$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r'}\Big|_\infty^r$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

7/8/20

(1)

The Potential of a localized charge distribution.

$\vec{E}$: The electric field is $E = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{1}{r^2}\right)\hat{r}$ and

$$dl = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

so,

$$E \cdot dl = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,dr.$$

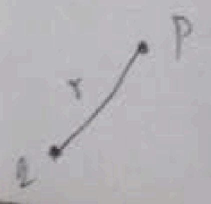
setting the reference point at infinity, the potential of a point charge q at the origin ~~is~~ is

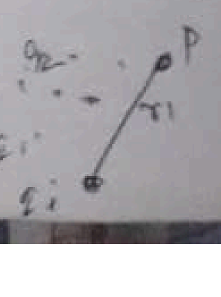
$$V(r) = -\int_{0}^{r} E \cdot dl$$

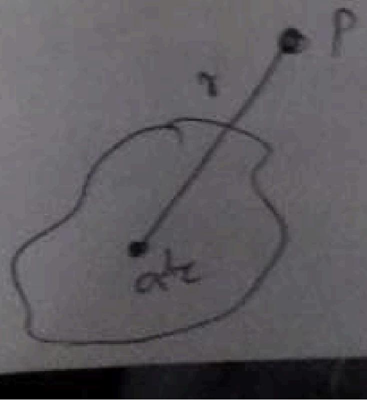
$$= -\frac{1}{4\pi\epsilon_0}\int_{\infty}^{r}\frac{q}{r'^2}\,dr'$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r'}\Big|_{\infty}^{r}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$



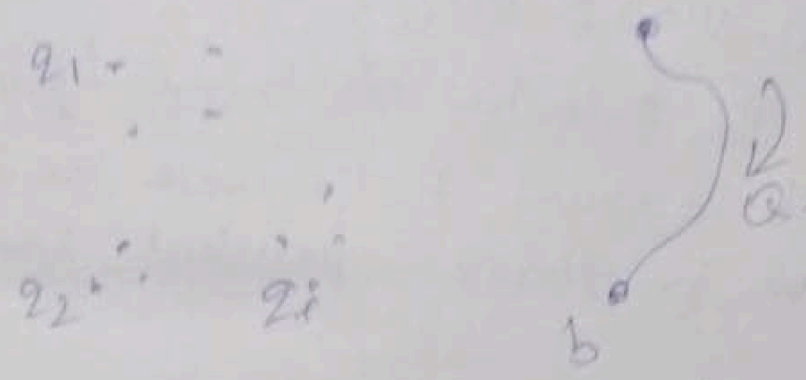




Work and energy in electrostatics

(7)

The work it takes to move a charge



At any point along the path, the electric force on Q is $F = QE$

∴ in opposition to this electrical force is $-QE$

Gravity exerts a force mg downwards but ~~you~~ we exert a force mg upward, of course ~~you~~ we could apply an even greater force – then the brick would accelerate and part of your effort would be "wasted" generated K.E. (kinetic energy)

The work do this

$$W = \int_a^b F \cdot dl = -Q\int_a^b E \cdot dl$$

$$= Q\left[V(b) - V(a)\right]$$

So as the height of pill-box Δh tends to zero the term arising from the bulk charge densities ρ_1, ρ_2 becomes negligible.

The integral form of Gauss's law then follows that

$$(D_2 \cdot \hat{n})\Delta A - (D_1 \cdot \hat{n})\Delta A \doteq \sigma \Delta A$$

which becomes exact in the limit $\Delta A \to 0$ when

$$(D_2 - D_1) \cdot \hat{n} = 0$$

∴ If there is no free surface charge the component of D normal to the interface is continuous.

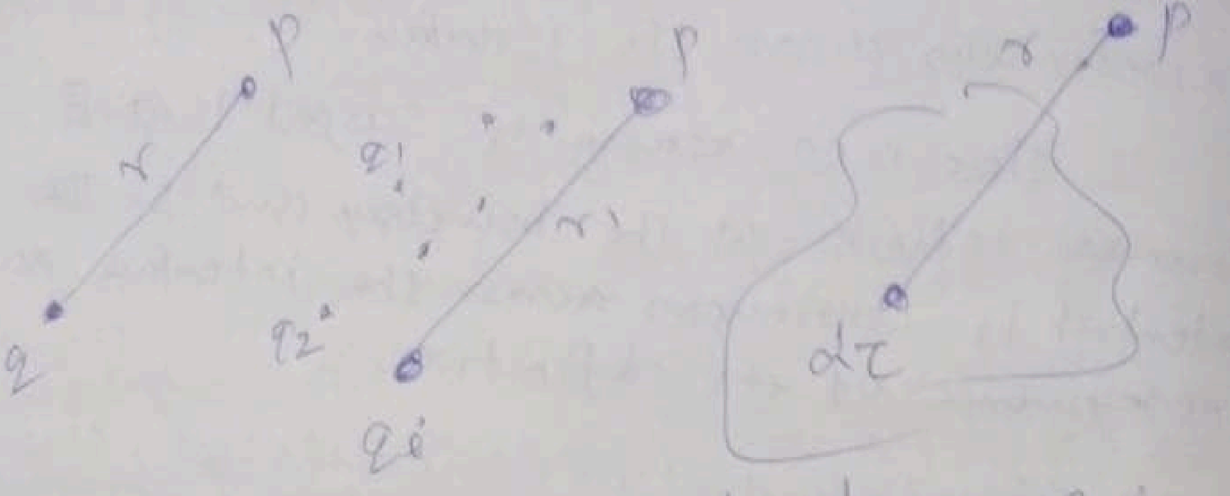
Since E is a conservative field and

$$\oint_{racs} E \cdot dl = 0 .$$

So $\Delta h \to 0$

$$(E_{2\parallel}\Delta l - E_{1\parallel}\Delta l) \to 0$$

~~There is no reason to suppose that E becomes infinite~~ at the



The potential of a point charge q is

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

where r, as always, is the distance from q to r. The potential of a collection of charges

$$V(r) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$$

for a continuous distribution

$$V(r) = \frac{1}{4\pi\epsilon_0} \int \frac{1}{r}\, dq.$$

In particular, for a volume charge, it's

$$V(r) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r')}{r}\, d\tau'$$

The solution of to poisson's eqn for a localized charge distribution.

The fig shows that the first charge q_1 takes no work, since there is no field yet to fight against.

Now bring in q_2,

$q_2 V_1(r_2)$ where V_1 is the potential due to q_1 and r_2 is the place we're putting q_1

$$W = \frac{1}{4\pi\epsilon_0} q_2 \left(\frac{q_1}{r_{12}}\right)$$

(r_{12} is the distance between q_1 and q_2 once they are in position)

Now bring in q_3, this requires work $q_3 V_{1,2}(r_3)$ where $V_{1,2}$ is the potential due to charges q_1 and q_2 namely

$$\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$$

Thus,

$$W_3 = \frac{1}{4\pi\epsilon_0} q_3 \left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$$

Illy the extra work to bring in q_4 will be

$$W_4 = \frac{1}{4\pi\epsilon_0} q_4 \left(\frac{q_1}{r_{14}} + \frac{q_2}{r_{24}} + \frac{q_3}{r_{34}}\right)$$

where each of the components is a function of x and y, putting this into maxwells eqn (iii) and (iv) we obtain

(i) $$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\omega B_z \quad \text{— (iv)}$$

(ii) $$\frac{\partial E_z}{\partial y} - ikE_y = i\omega B_x \quad \text{— (v)}$$

(iii) $$ikE_x - \frac{\partial E_z}{\partial x} = i\omega B_y \quad \text{— (vi)}$$

eqn (ii) and (iii) (v) and (vi) can be solved for E_x, E_y, B_x and B_y

(i) $$E_x = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y}\right),$$

(ii) $$E_y = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial y} - \omega\frac{\partial B_z}{\partial x}\right)$$

(iii) $$B_x = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial x} - \frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right)$$

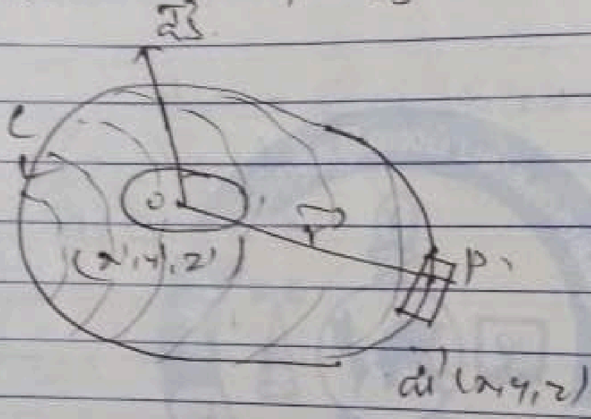
(iv) $$B_y = \frac{i}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial y} + \frac{\omega}{c^2}\frac{\partial E_z}{\partial x}\right)$$

Ampere's Circuit Law (curl B)

According to it for steady current the line integral of magnetic induction vector B around a closed path is equal to μ_0 times the total current crossing any surface bounded by the line integral path

i.e.)

$$\oint B \cdot dl = \mu_0 \int_s J \cdot ds = \mu_0 I.$$



To obtain this law consider a unit element $J \cdot ds$ and a closed path C as shown in fig. The element of induction at P due to current at O by Biot–Savart law will be.

$$dB = \frac{\mu_0}{4\pi} \frac{J' \times r}{r^3}$$

with $J' = J_{(x, y, z)}$ and

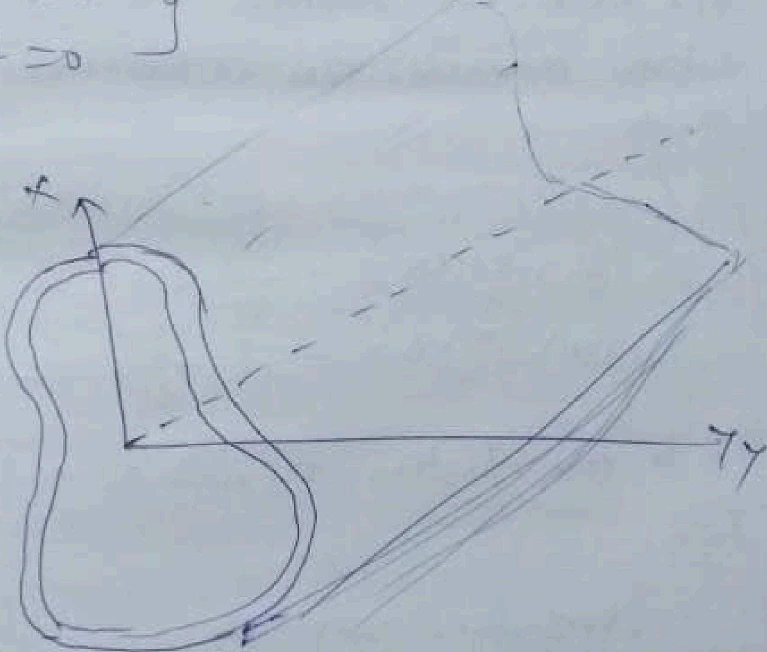
$$r^2 = (x-x')^2 + (y-y')^2 + (z-z')^2$$

Wave Guides.

Electromagnetic waves confined to the interior of a hollow pipe, or, wave guide, we will assume the wave guide is a perfect conductor, so that $E=0$ and $B=0$ inside the material itself, and hence the boundary conditions at the inner wall are

(i) $E^{\parallel}=0$

(ii) $E^{\perp}=0$



Free charges and currents will be induced (59)
On the surface is such a way as to enforce these constraints we are interested in monochromatic wave that propagate down the tube, so E and B have the general form.

$$\left.\begin{aligned} &(i)\ \vec{E}(x,y,z,t) = \vec{E_0}(x,y)\, e^{i(kz-\omega t)} \\ &(ii)\ \vec{B}(x,y,z,t) = \vec{B_0}(x,y)\, e^{i(kz-\omega t)} \end{aligned}\right\} -$$

The electric and magnetic fields must, of course, satisfy maxwell's eqns. in the interior of the wave guide,

$$\left.\begin{aligned} &(i)\ \nabla\cdot E = 0 \quad (ii)\ \nabla\cdot B = 0 \\ &(iii)\ \nabla\times E = -\frac{\partial B}{\partial t} \\ &(iv)\ \nabla\times B = -\frac{1}{c^2}\frac{\partial E}{\partial t} \end{aligned}\right\}$$

is to find functions $\vec{E_0}$ and $\vec{B_0}$ such that the field,

longitudinal components

$$\vec{E_0} = E_x\hat{x} + E_y\hat{y} + E_z\hat{z}, \quad \vec{B_0} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$

$$\nabla^2\phi = \frac{1}{4\pi\epsilon_0}\int \rho(x)\left[-4\pi\,\delta(x-x')\right]dv'$$

$$\boxed{\nabla^2\phi = -\frac{\rho(x)}{\epsilon_0}}$$

which is exactly poisson's eqn.

Ex show that the potential function $\phi = q(x^2+y^2+z^2)^{-1/2}$ satisfies the laplace eqn.

Solution we have $\phi = q(x^2+y^2+z^2)^{-1/2}$

$$\frac{\partial\phi}{\partial x} = \frac{\partial}{\partial x}\left[q(x^2+y^2+z^2)^{-1/2}\right]$$

$$= -qx(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2\phi}{\partial x^2} = -qx\left(-\frac{3}{2}\right)(x^2+y^2+z^2)^{-5/2}\,2x - q(x^2+y^2+z^2)^{-3/2}$$

$$= q\left[\frac{3x^2}{(x^2+y^2+z^2)^{5/2}} - \frac{1}{(x^2+y^2+z^2)^{3/2}}\right] \quad \text{—(1)}$$

11ly

$$\frac{\partial^2\phi}{\partial y^2} = q\left[\frac{3y^2}{(x^2+y^2+z^2)^{5/2}} - \frac{1}{(x^2+y^2+z^2)^{3/2}}\right] \quad \text{—(2)}$$

and

$$\frac{\partial^2\phi}{\partial z^2} = q\left[\frac{3z^2}{(x^2+y^2+z^2)^{5/2}} - \frac{1}{(x^2+y^2+z^2)^{3/2}}\right] \quad \text{—(3)}$$

(1) + (2) & + (3).

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = q\left[\frac{3(x^2+y^2+z^2)}{(x^2+y^2+z^2)^{5/2}} - \frac{3}{(x^2+y^2+z^2)^{3/2}}\right]$$

Suppose a point charge q is held a distance d above an infinite grounded conducting plane. We can find out the potential in the region above the plane. Forget about the actual problem we are going to study a complete different situation. The new problem consists of two point charges $+q$ at $(0,0,d)$ and $-q$ at $(0,0,-d)$ and no conducting plane.

For this configuration we can easily write down the potential

$$V(x,y,z) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{\sqrt{x^2+y^2+(z-d)^2}} - \frac{q}{\sqrt{x^2+y^2+(z+d)^2}}\right]$$

The denominators represent the distance from (x,y,z) to the charges $+q$ and $-q$ respectively.

1. $V=0$ when $z=0$ and
2. $V \to 0$ for $x^2+y^2+z^2 \gg d^2$. and the only charge in the region $z>0$ is the point charge $+q$ at $(0,0,d)$. Thus the second configuration produces exactly the same potential as the first configuration in the upper region $z \geq 0$.

The answer is independent of the path we take from a to b. in mechanics it's called electrostatic force, "conservative". Dividing through by Q

$$V(b) - V(a) = W/Q.$$

In words. The potential difference b/w points a and b is equal to the work per unit charge required to carry a particle from a to b

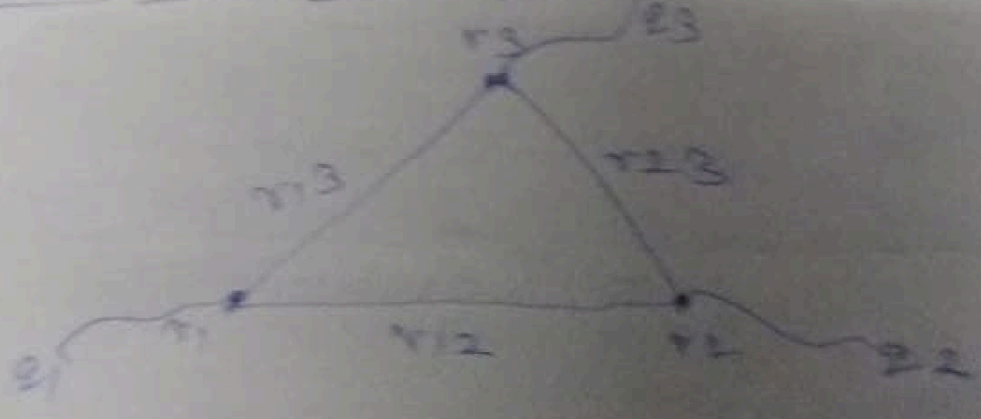
Q in form far away and stick it at point r_1. The work must do is

$$\boxed{W = Q[V(r) - V(\infty)]}$$

The reference point at infinity.

$$\boxed{W = Q\,V(r)}$$

<u>THE ENERGY OF A POINT CHARGE DISTRIBUTION</u>



The total work necessary to assemble to the first four charges, is

$$W = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_1 q_4}{r_{14}} + \frac{q_2 q_3}{r_{23}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right)$$

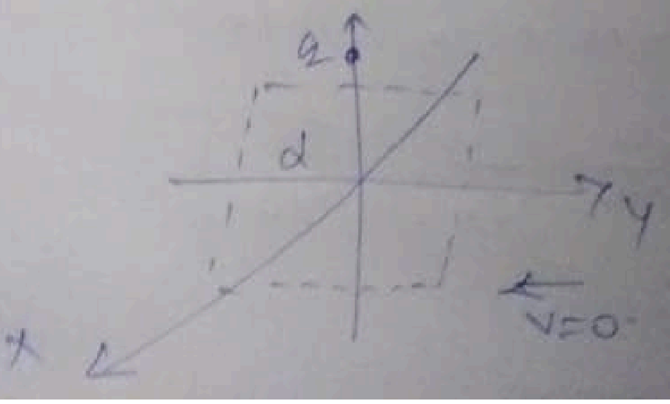
$$W = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \sum_{j>i}^{n} \frac{q_i q_j}{r_{ij}}$$

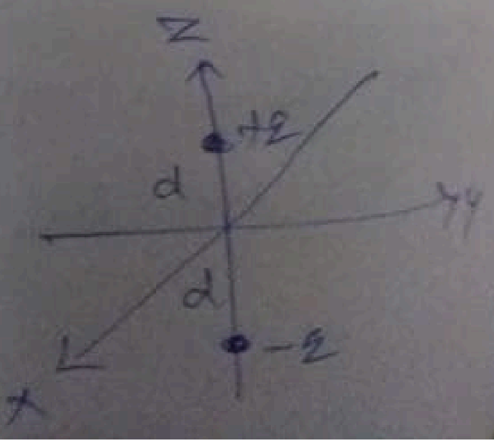
$$\boxed{\therefore W = \frac{1}{2} \sum_{i=1}^{n} q_i V(r_i)}$$

=x=

Image Problems

Classical Image Problem.





Now $\int_a^b \vec{E}\cdot d\vec{l} = \frac{1}{4\pi\varepsilon_0} q \int_a^b \frac{dr}{r^2}$

$$= \frac{1}{4\pi\varepsilon_0} q \left(\frac{-1}{r}\right)_{r_a}^{r_b}$$

$$\int_a^b \vec{E}\cdot d\vec{l} = \frac{1}{4\pi\varepsilon_0} q \left(-\frac{1}{r_b} + \frac{1}{r_a}\right)$$

$$\int_a^b \vec{E}\cdot d\vec{l} = \frac{1}{4\pi\varepsilon_0} q \left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

Then $r_a = r_b$.

$$\oint \vec{E}\cdot d\vec{l} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r_a} \cancel{(1-1)} \; (0)$$

$\Rightarrow \oint \vec{E}\cdot d\vec{l} = 0$.

by stokes theorem. $\oint (\vec{\nabla}\times\vec{E})\, da = \oint \vec{E}\, d\vec{l} = 0$

$$\boxed{\vec{\nabla}\times\vec{E} = 0}$$

Curl of E is zero. //.

(2)

Question No.

$$= \frac{1}{4\pi\epsilon_0}\int \rho(r')\, \nabla^2\left(\frac{1}{|r-r'|}\right) dv \quad — (7)$$

now calculate $\nabla^2\left(\frac{1}{|r-r'|}\right)$ transform to $\nabla^2\left(\frac{1}{r}\right)$

By direct calculation we find that $\nabla^2\left(\frac{1}{r}\right) = 0$ for $r \neq 0$.

At $r = 0$, however the expression is undefined.

we integrate $\nabla^2\left(\frac{1}{r}\right)$ over a small volume V containing the origin. Then we use the divergence theorem to obtain ~~the~~ a surface integral.

$$\int_V \nabla^2\left(\frac{1}{r}\right) dv = \int_V \nabla\cdot\nabla\left(\frac{1}{r}\right) dv$$

$$= \int_S \nabla\left(\frac{1}{r}\right)\cdot n\, da$$

$$= \int_S \frac{\partial}{\partial r}\left[\frac{1}{r}\right] r^2 d\Omega$$

$$= -4\pi$$

Thus we have $\nabla^2(1/r) = 0$ for $r \neq 0$. and that its volume integral is -4π. consequently we may write improper equation.

$$\nabla^2\left(\frac{1}{r}\right) = -4\pi\, \delta(r) \text{ or}$$

more generally

$$\nabla^2\left(\frac{1}{|r-r'|}\right) = -4\pi\, \delta(r-r')$$

curl E — curl means in simple manner. 06/8/20

$\nabla \times E = 0$

The curl of electric field is not always zero. The general law for the curl of the electric field is one of the maxwell's equations which states that

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ It is only when the magnetic field is either zero (or) constant so it true that the curl of electric field is zero.

Now, Electric field due to a point charge of a distance r is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

consider the point charge is placed at the centre on it origin.

Now in spherical co-ordinates system

$d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$

Now $\vec{E}\cdot d\vec{l} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,dr.$

Question No.

Poisson's and Laplace's equations (1)

The fundamental differential equations which must be satisfied by the potential ϕ and develop various alternative methods to solve electrostatic problem.

* Poisson's and Laplace's equation.

The Gauss's theorem in differential form is

$$\text{div } E = \rho/\epsilon_0 \text{ (in free space)} \quad \text{—} (1)$$

In a purely electrostatic field

$$E = -\text{grad } \phi \quad \text{—} (2)$$

combining (1) & (2) we obtain.

$$\text{div grad } \phi = -\rho/\epsilon_0 \quad \text{—} (3)$$

$$\nabla \cdot \nabla \phi = -\rho/\epsilon_0$$

ie) $$\nabla^2 \phi = -\rho/\epsilon_0 \quad \text{—} (4)$$

The operator $\nabla \cdot \nabla = \nabla^2$ is known as the Laplacian and eqn (4) is known as poisson's eqn.

In the charge free region $\rho = 0$ hence poisson's eqn charge free region takes the form

$$\nabla^2 \phi = 0 \quad \text{—} (5)$$

This eqn is known as ~~Poissions~~ Laplace's eqn.

The scalar potential ϕ for an arbitrary charge distribution over a given volume is

$$\phi(r) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r')}{|r-r'|} \, dv' \quad \text{—} (6)$$

where $\rho(r')$ is the charge density at r' and r is the point of observation.

$$\nabla^2 \phi = \frac{1}{4\pi\epsilon_0} \nabla^2 \int \frac{\rho(r')}{|r-r'|} \, dv'$$

$$\vec{E}_{0I} - \vec{E}_{0R} = \beta \vec{E}_{0T}$$

$$\beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$$

in terms of incident amplitude

$$\vec{E}_{0R} = \left(\frac{1-\beta}{1+\beta}\right)\vec{E}_{0I},$$

$$\vec{E}_{0T} = \left(\frac{2}{1+\beta}\right)\vec{E}_{0I}$$

If the permittivities μ are close to their values in vacuum then $\beta = \frac{v_1}{v_2}$ and we have

$$\vec{E}_{0R} = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)\vec{E}_{0I}$$

$$\vec{E}_{0T} = \left(\frac{2v_2}{v_2 + v_1}\right)\vec{E}_{0I} \qquad v_2 < v_1$$

$$v_2 > v_1$$

$$E_{0R} = \left|\frac{v_2 - v_1}{v_2 + v_1}\right| E_{0I}, \quad E_{0T} = \left(\frac{2v_2}{v_2 + v_1}\right) E_{0I}$$

in terms of the indices of refraction

$$E_{0R} = \left|\frac{n_1 - n_2}{n_1 + n}\right| E_{0I}, \quad E_{0T} = \left(\frac{2n_1}{n_1 + n_2}\right) E_{0I}$$

Which travels back to the left in medium (I)

and a transmitte wave

$$\left.\begin{aligned} \bar{E}_T(z,t) &= \bar{E}_{0T}\, e^{i(k_2 z - \omega t)}\, \hat{x} \\ \bar{B}_T(z,t) &= \frac{1}{v_2} \bar{E}_{0T}\, e^{i(k_2 z - \omega t)}\, \hat{y} \end{aligned}\right\} -$$

Which continues on the right in medium.

Note the minus sign in $\vec{B}_R$ as required by the fact that the Poynting vector aims in the direction of propagation.

At $z = 0$ the combined fields on the left $\vec{E}_I + \vec{E}_R$ and $\bar{B}_I + \bar{B}_R$ must join the fields on the right, $\bar{E}_T$ and $\overline{B_T}$ in accordance with the boundary conditions.

In this case there are no components perpendicular to the surface, so (i) (ii) are trivial. However (iii) requires that $\vec{E}_{0I} + \bar{E}_{0R} = \bar{E}_{0T}$

while (iv) says

$$\frac{1}{\mu_1}\left(\frac{1}{v_1}\bar{E}_{0I} - \frac{1}{v_1}\vec{E}_{0R}\right) = \frac{1}{\mu_2}\left(\frac{1}{v_2}\vec{E}_{0T}\right)$$

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{c}{n}.$$

$$n \equiv \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}}$$

is the index of refraction of the material.

For most materials, μ is very close to μ_0, so

$$n \cong \sqrt{\epsilon_r}.$$

where ϵ_r is the dielectric constant since ϵ_r is ~~always~~ almost alway greater than 1, light travels more slowly through matter, - a fact that is well known from optics

with the simple transcription $\epsilon_0 \to \epsilon$, $\mu_0 \to \mu$, and hence $c \to v$ the energy density is

$$u = \frac{1}{2}\left(\epsilon E^2 + \frac{1}{\mu} B^2\right)$$

and the Poynting vector is

$$S = \frac{1}{\mu}(E \times B)$$

(32)

It suffices, then, to determine the longitudinal components E_z and B_z, if we knew those, we could quickly calculate all the others, just by differentiating, The remaining maxwell equations yields ~~unequal~~ uncoupled eqn for E_z and B_z :

$$\text{(i)} \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + (\omega/c)^2 - k^2\right] E_z = 0.$$

$$\text{(ii)} \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + (\omega/c)^2 - k^2\right] B_z = 0.$$

if $E_z = 0$, we call these TE (Transverse electric) waves if $B_z = 0$ they are called TM ("transverse magnetic") waves if both $E_z = 0$ and $B_z = 0$ we call them TEM waves It turns out that TEM waves cannot occur in a hollow wave guid.

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Electromagnetic waves in matter.

Propagation in Linear media:

maxwell's equation becomes

$$\left.\begin{array}{l} \text{(i) } \nabla\cdot D = 0 \\ \text{(ii) } \nabla\cdot B = 0 \\ \text{(iii) } \nabla\times E = -\dfrac{\partial B}{\partial t} \\ \text{(iv) } \nabla\times H = \dfrac{\partial D}{\partial t} \end{array}\right\}$$ If the medium is linear

$$D = \epsilon E, \quad H = \frac{1}{\mu} B.$$

and homogeneous (so ϵ and μ do not vary from point to point) maxwell's eqns reduce to

(i) $\nabla\cdot E = 0$ (ii) $\nabla\cdot B = 0$ (iii) $\nabla\times E = -\dfrac{\partial B}{\partial t}$

(iv) $\nabla\times B = \cancel{\mu_0}\, \mu\epsilon \dfrac{\partial E}{\partial t}$

Only in the replacement of $\mu\epsilon_0$ by $\mu\epsilon$

Evidently electromagnetic waves propagate through a linear homogeneous medium at a speed

The potential of a point charge q is

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

Where r, as always is the distance from q to r

The potential of a collection of charges is

$$V(r) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$$

for a continuous distribution,

$$V(r) = \frac{1}{4\pi\epsilon_0} \int \frac{1}{r} \, dq.$$

In particular, for a volume charge it's

$$V(r) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r')}{r} \, d\tau'$$

The solution to poisson's eqn for a localized charge distribution.

" The amount of induced voltage is equal to the rate of change of the magnetic flux. This can be represented in eqn form.

$$\xi = \frac{\Delta \phi}{\Delta t}$$

— x —

(1) Faraday's Laws of Electromagnetic Induction.

1. whenever the magnetic flux linked with a closed circuit changes, an induced e.m.f is set up the circuit whose magnitude at any instant is proportional to the rate of change of magnetic flux linked with the circuit.

If ξ is the magnetic flux linked with the circuit at any instant t and e is the induced e.m.f. Then

$$e \propto \frac{d\phi}{dt}.$$

(or)

$$\xi = -\frac{d\Phi_m}{dt}$$

$$\Delta V = V_f - V_i$$
$$= -\int_i^f \vec{E} \cdot \vec{ds}$$

let's calculate ΔV (or) $\mathcal{E}$ (emf) over the closed loop $\Delta V = \mathcal{E} = \oint \vec{E} \cdot \vec{ds}$

From the other side, Faraday's law:

$$\mathcal{E} = -\frac{d\Phi_m}{dt}$$

$$\oint \vec{E} \cdot \vec{ds} = -\frac{d\Phi_m}{dt}$$ This law implies that a changing magnetic flux will induce an induced electric field.

2e **Two types of the electric field**

when $A = \text{const}$ and $\theta = 0$.

$$\Phi_m = \vec{B} \cdot \vec{A} = BA \Rightarrow \oint \vec{E} \cdot \vec{ds} = A \left[\frac{dB}{dt}\right]$$

3 Inductance

From Faraday's law of induction, "any change in magnetic field through a circuit induces an electromotive force (EMF) (voltage) in the conductors. a process known as electromagnetic induction".

Inductance is defined as the ratio of the induced voltage to the rate of change of current causing it.

Faraday's law of induction makes use of the magnetic flux Φ_B through a region of space enclosed by a wire loop. The magnetic flux is defined by a surface integral.

$$\Phi_B = \int_{\Sigma} B \cdot dA .$$

Where dA is an element of the surface Σ enclosed by the wire loop. B is the magnetic field, the dot product $B \cdot dA$ corresponds to an infinitesimal amount of magnetic flux.

Question No.

Comparing eqn (a) and (b) we find that

$dF_{12} \neq -dF_{21}$

from Newton's third law:

$dF_{12} = -dF_{21}$.

$F_{12} = \oint_1 dF_{12}$

$$F_{12} = \mu_0/4\pi \, I_1 I_2 \oint_1 \oint_2 \frac{(dl_1 \cdot r)\, dl_2 - (dl_1 \cdot dl_2)}{r^3}$$

Now as.

$$\oint_1 \oint_2 \frac{(dl_2 \cdot r)}{r^3} dl_1 = \oint_2 dl_2 \int_1 \frac{(dl_1 \cdot r)}{r^3}$$

$$= \oint_2 dl_2 \int_1 \left[- \nabla (1/r) \right] dl_1 \quad \left[\text{as } \nabla \left(\frac{1}{r}\right) = -\frac{r}{r^3} \right]$$

$$= -\oint_2 dl_2 \text{ curl grad } (1/r) \, ds_1.$$

as $\oint A \cdot dl = \int_s \text{curl} A \, ds$.

$= 0$ [as curl grad of a scalar $= 0$]

$$F_{21} = -\frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \oint_2 \frac{(dl_1 \cdot dl_2) \cdot r}{r^3} \quad \text{— (c)}$$

Hence.

$$F_{12} = \frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \oint_2 \frac{(dl_2 \cdot dl_1) r - (dl_2 \cdot r) dl_1}{r^3}$$

or) $$F_{12} = \frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \oint_2 \frac{(dl_2 \cdot dl_1) r}{r^3} \quad \text{— (d)}$$

$$\oint_1 \oint_2 \frac{(dl_2 \cdot r)\, dl_1}{r^3} = -\oint_1 dl_1 \oint_2 dl_2 \cdot \text{grad}(1/r)$$

$= -\oint_1 dl_1 \text{ curl grad } (1/r) \cdot ds_2 = 0$

$F_{12} = -F_{21}$. //

Question No.

unit III Electromagnetic Induction

Faraday's law of Induction is a basic law of electromagnetism "predicting how a magnetic field will interact with an electric circuit to produce an electromagnetic force (Emf) a phenomenon called electromagnetic Induction.

Faraday's laws.

Ist law.

Any change in the magnetic field of a coil of wire will cause an emf to be induced in the coil. This emf induced is called induced emf and if the conductor circuit is closed, the current will also circulate through the circuit and this current is called induced current.

IInd law.

The magnitude of emf induced in the coil is equal to the rate of change of flux that linkages with the coil. The flux linkage of the coil is the product of number of turns in the coil and flux associated with the coil.

$$T_1 = N\phi_1 \text{ Wb}$$

$$T_2 = N\phi_2 \text{ Wb}$$

change in flux linkage $N(\phi_2 - \phi_1)$

Unit II

Magnetostatics.

Is the study of magnetic fields in systems where the currents are steady

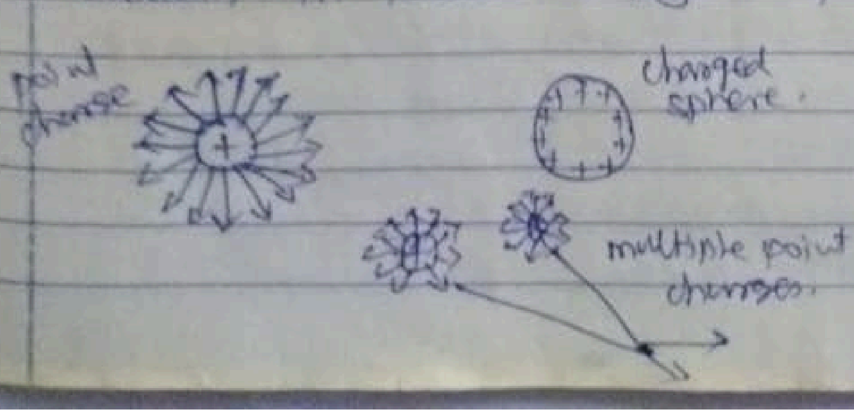
The Lorentz force law.

The magnetic field B is defined from the Lorentz force law and specifically from the magnetic force on a moving charge. The implications of this expression include.
1. The force is perpendicular to both the velocity v of the charge q and the magnetic field B

$$\vec{F} = \underbrace{q\vec{E}}_{\text{electric force}} + \underbrace{q\vec{v} \times \vec{B}}_{\text{magnetic force}}$$

Electric field.

Electric field is defined as the electric force per unit charge. The direction of the field is taken to be the direction of the force it would exert on a positive test charge. The electric field is radially outward from a positive charge and radially in toward a negative point charge.



While the II term in eqn (3) ie

$$\frac{\mu_0}{4\pi} \nabla \int \nabla \cdot (J'/r) d\tau'$$

$$= \frac{\mu_0}{4\pi} \nabla \int \left[\frac{1}{r} \nabla \cdot J' + J' \cdot \nabla (1/r) \right] d\tau'$$

(as $\nabla \cdot (\psi V) = \psi \nabla \cdot V + V \cdot \nabla \psi$)

$$= \frac{\mu_0}{4\pi} \nabla \int J' \cdot \nabla (1/r) d\tau'$$

as $\nabla \cdot J' = 0$ as J' is not a (function of x, y and z)

$$= -\frac{\mu_0}{4\pi} \nabla \int \nabla' \cdot (J'/r) d\tau'$$

$$= \frac{\mu_0}{4\pi} \nabla \oint (J'/r) \cdot ds$$

$$\therefore \frac{\mu_0}{4\pi} \nabla \int \nabla \cdot (J'/r) d\tau' = 0 \quad — (8)$$

Sub eq (8) and (6) in (3) we get

$$\nabla \times B = \mu_0 J \quad — (9)$$

$$\int (\nabla \times B) \cdot ds = \mu_0 \int_s J \cdot ds \quad — (5)$$

Stoke's Theorem

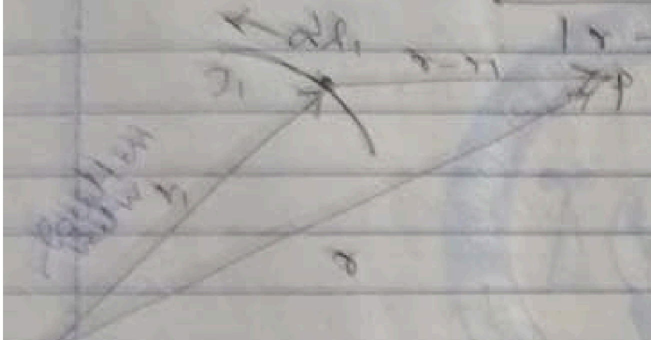
$$\int_s (\nabla \times B) \cdot ds = \oint_c B \cdot dl \quad — (7)$$

Biot savart law.

The magnetic flux density B to the current and also the law of force between one current and another.

(i) Let dl_1 be an element of length of a wire with its sense taken in the direction of the current I_1 flowing in the wire. The magnetic flux density dB due to this element at a point P specified by the location vector r is given in magnitude and direction by.

$$dB = K \frac{I_1 dl_1 \times (r - r_1)}{|r - r_1|^3} \quad \text{--- (1)}$$



where r_1 is the location vector of the element, dl.

This means the flux density is directly proportional to the current flowing in the circuit and to the length of the element of the wire and inversely proportional to the square of the distance of the point from the element of the wire.

Comparison of electrostatics and magnetostatics

Electrostatics		Magnetostatics
1. stationary electric charge		Element carrying steady current
2. coulomb's law	–	Biot and Savart's law
3. $E = F/q$	–	$F = q(V \times B)$
4. $E = -\nabla\phi$.	–	$B = \nabla \times A$.
5. Gauss's theorem $\Phi_E = \oint_S E\,dA = \frac{Q_s}{\epsilon_0}$	–	Ampere's law $\oint B \cdot dl = \int_S \nabla \times B \cdot ds =$
6. Electric scalar potential	–	magnetic scalar potential & magnetic vector potential
7. Poisson eqn.		eqn similar to Poisson's $\nabla^2 A = -\mu_0 j$

Question No.

$$\boxed{\frac{r}{r^3} = -\nabla\frac{1}{r}}$$

so that

$$B = \frac{\mu_0}{4\pi}\int \frac{J' \times r}{r^3}\, d\tau'$$

ie $B = \frac{\mu_0}{4\pi}\int J' \times \nabla\left(\frac{-1}{r}\right) d\tau' \quad \left(\nabla\frac{1}{r} = r/r^3\right)$

But as curl SV = S curl V − V × grad S

curl $\frac{J'}{r} = \frac{1}{r}$ curl $J' - J' \times \nabla(1/r)$

[since $\nabla \times J' = 0$ as J' is not a function of x, y, and z)

$\therefore B = \mu_0/4\pi \int \nabla \times (J'/r)\, d\tau'$

So, $\nabla \times B = \frac{\mu_0}{4\pi}\int \nabla \times \nabla \times \left(\frac{J'}{r}\right) d\tau'$

Now as $\nabla \times \nabla \times V$ = grad div V − $\nabla^2 V$.

$$\nabla \times B = \frac{\mu_0}{4\pi}\int \left[\nabla\left\{\nabla\cdot\left(\frac{J'}{r}\right)\right\} - \nabla^2(J'/r)\right] d\tau'$$

ie $\nabla \times B = \frac{\mu_0}{4\pi}\int -\nabla^2(J'/r)\, d\tau' + \frac{\mu_0}{4\pi}\nabla\int \nabla\cdot(J'/r)\, d\tau'$ —— (1)

$\therefore -\frac{\mu_0}{4\pi}\int \nabla^2(J'/r)\, d\tau' = -\frac{\mu_0}{4\pi}\int J'\nabla^2(1/r)\, d\tau'$.

as J' is not a function of x, y, z

$= \frac{-\mu_0}{4\pi}\int J'\left(-4\pi\delta(r - r')\right) d\tau'$.

[as $\nabla^2(1/r) = -4\pi\delta(r - r')$]

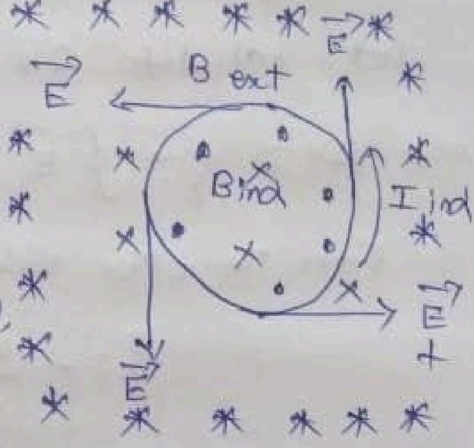
$= \mu_0 J'$. —— (2)

(2) Induced Electric field.

Consider a conducting loop in an increasing uniform magnetic field,

$B_{ext} \nearrow \Rightarrow \Phi_B \nearrow \Rightarrow B_{ext} \uparrow \downarrow B_{ind} \Rightarrow$

I_{ind} ccw.



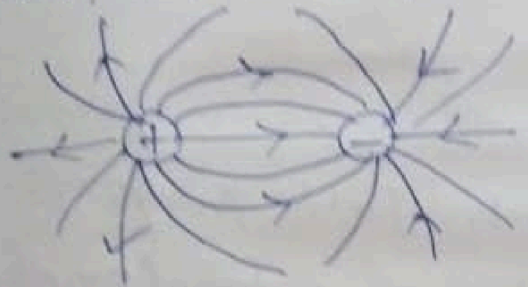
* So, according to Lenz's law, there is an induced current in the counterclockwise direction

* If there is a current, there must be something that acts on the charge carriers to make them move.

* So we infer there must be an induced electric field tangent to the loop at all points

* The conducting loop is not necessary to generate E

* The loop was used as a probe system to convince ourselves that there was the E field.

Coulomb electric field	Induced electric field
1. E is produced by charges.	1. E is produced by changing B, not by charges
2. Coulomb electric field lines start / stop on charges. 	2. Induced electric field lines form closed loop.
3. Induced E is conservative. $\mathcal{E} = \oint \vec{E} \cdot \vec{ds} = 0$	3. Induced E is non-conservative. $\mathcal{E} = \oint \vec{E} \cdot \vec{ds} = -\frac{d\Phi_m}{dt}$

Faraday's law

$$\mathcal{E} = \oint \vec{E} \cdot \vec{ds} = -\frac{d\Phi_m}{dt}.$$

A changing magnetic field creates an induced electric field.

Lenz's law

$$E = -N\left(\frac{d\Phi}{dt}\right) \text{ volts.}$$

unit-III

19/8/2020 (1)

Electromagnetic Induction.

Electromagnetic induction is the process a "current can be induced to flow due to a changing magnetic field".

∴ The force on a current - carrying wire due to the electrons which move within it when a magnetic field is present is a classical example.

" This process also works in reverse. Either moving a wire through a magnetic field (or) (Equivalently) changing the strength of the magnetic field over time can cause a current to flow.

1. Faraday's law

The rate of change of magnetic flux through a loop to the magnitude of the electro-motive force ε induced in the loop. The relationship is

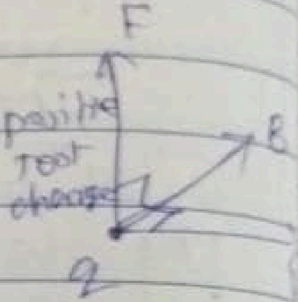
$$\varepsilon = \frac{d\Phi}{dt}$$

The electromotive force (or) EMF refers to the potential difference across the unloaded loop. (i.e) when the resistance in the circuit is high)

Electric field in newtons

$\vec{E} = \frac{\vec{F}}{q}$ electric force in newtons
charge in coulombs.

$\therefore \vec{F} = \underbrace{q\vec{E}}_{\text{Electric force}} + \underbrace{q\vec{v} \times \vec{B}}_{\text{magnetic force.}}$



Lorentz Force Law.

magnetic field

magnetic field are produced by electric currents, which can be macroscopic currents in wires - (or) microscopic currents associated with electrons in automic orbits.

The magnetic field B is defined in terms of force on moving charge in the Lorentz force Law.

The resistance of the wire is $R = \cancel{\rho/A}\ \rho l/A$

For copper $\rho = 1.72 \times 10^{-8}\ \Omega m$

The cross-sectional area of the wire is

$A = \pi r^2$

$= \pi(0.0005)^2 = 7.85 \times 10^{-7}\ m^2$

The resistance of the wire then is $((1.72 \times 10^{-8}) \times 160/(7.85 \times 10^{-7}))\ \Omega = 3.5\ \Omega$

The current is $I = V/R$

$= (1.5/3.5)\ A$

$= 0.428\ A$

Induced surface Charge

Unit I

The surface charge density σ induced on the conductor surface is

$$\sigma = -\varepsilon_0 \frac{\partial V}{\partial z}\Big|_{z=0}$$

$$\Rightarrow \sigma(x,y) = \frac{-qd}{2\pi (x^2+y^2+d^2)^{3/2}}$$

As expected, The induced charge is negative (assuming q is positive) and greatest at $x = y = 0$

The total induced charge $\boxed{Q = \int \sigma \, da = -q}$

Force and Energy

The charge q is attracted towards the plane, because of the ~~neg~~ negative induced surface charge.

The force $\vec{F} = -\frac{1}{4\pi\varepsilon_0} \frac{q^2}{(2d)^2} \hat{z}$

One can determine the energy by calculating the work required to bring q in from infinity.

$$W = \int_{\infty}^{d} \vec{F} \cdot d\vec{l} = \frac{1}{4\pi\varepsilon_0} \int_{\infty}^{d} \frac{q^2}{4z^2} dz =$$

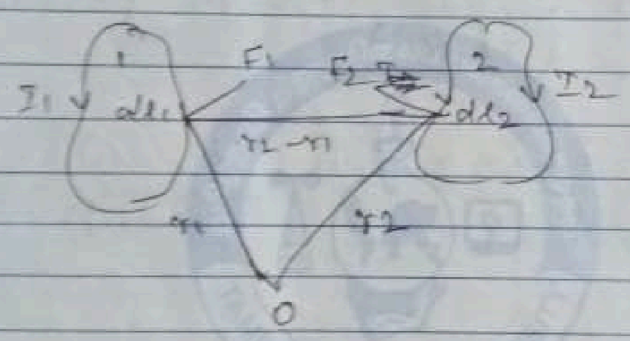
(iii) If there are two closed circuits

The force on circuit 2 due to circuit 1 is

$$F_2 = \oint I_2\, dl_2 \times B_{21}$$

where B_{21} is the magnetic flux density due to circuit 1 at the position where circuit 2 is located. We assume B_{21} to be constant over the region occupied by the circuit.

$$\therefore F_2 = \oint_2 I_2 dl_2 \times \frac{\mu_0}{4\pi} \oint \frac{I_1 dl_1 \times (r_2 - r_1)}{|r_2 - r_1|^3}$$



$$F_2 = \frac{\mu_0}{4\pi} I_1 I_2 \oint_2 \oint_1 \frac{dl_2 \times [dl_1 \times (r_2 - r_1)]}{|r_2 - r_1|^3}$$

Here the force exerted by the circuit 2 on the circuit 1 is

$$F_1 = \frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \oint_2 \frac{dl_1 \times [dl_2 \times (r_1 - r_2)]}{|r_1 - r_2|^3}$$

$$\frac{dl_2 \times [dl_1 \times (r_2 - r_1)]}{|r_2 - r_1|^3} \neq \frac{dl_1 \times [dl_2 \times (r_1 - r_2)]}{|r_1 - r_2|^3}$$

This is the differential form of Ampere's circuital law and signifies that magnetic field is rotational.

So from (4) and (5)

$$\oint B \cdot dl = \mu_0 \int_s J \cdot ds = \mu_0 I \quad -(6)$$

Question No.

an inverse square law. The constant k depends upon the materials as well as on the system of units used. In free space, if the current is measured in e.s.u and the magnetic flux density in e.m.u.

$$K = 1/c$$

In S.I units K is taken to be equal to $\mu_0/4\pi$ to ensure compatibility between the experimental law

The constant μ_0 is termed the permeability of free space and its value is exactly $4\pi \times 10^{-7}\ NA^{-2}$

$$dB = \frac{\mu_0}{4\pi} \frac{I_1 dl_1 \times (r - r_1)}{|r - r_1|^3} \quad \text{— (2)}$$

$$B = \frac{\mu_0 I_1}{4\pi} \int \frac{dl_1 \times (r - r_1)}{|r - r_1|^3} \quad \text{— (3)}$$

Eqn (2) (3) are the statement of the Biot-Savart law.

Question No.

Steady currents

A current requires moving charges. Let $\rho_+ = n_+ q_+$ be the density of the positive charges in some region, i.e. the amount of positive charge per unit volume, and let $\rho_- = n_- q_-$ be the density of the negative charges. Here n_+ and n_- are the number of positively and negatively charged particles per unit volume and q_+ and q_- are the charge of each positively and negatively charged particles respectively.

The current density j is defined as $j = \rho_+ \langle v_+ \rangle + \rho_- \langle v_- \rangle$

problem:

A copper wire has a length of 160 m and a diameter of 1.00 mm. If the wire is connected to a 1.5V battery, how much current flows through the wire?

Solution: The current can be found from Ohm's law. $V = IR$

V is the battery voltage.
so if R can be determined,
the current can be calculated

$$F_2 \neq -F_1$$

$$\therefore F_2 = \frac{\mu_0}{4\pi} I_1 I_2 \oint_2 \oint_1 \left[\frac{[(dl_2 \cdot (r_2 - r_1))] \, dl_1}{|r_2 - r_1|^3} - \frac{(dl_1 \cdot dl_2)(r_2 - r_1)}{|r_2 - r_1|^3} \right]$$

the first integral

$$\oint_2 \frac{dl_2 (r_2 - r_1)}{|r_2 - r_1|^3} = -\oint \nabla_2 \left(\frac{1}{|r_2 - r_1|} \right) \cdot dl_2 = 0$$

$$\therefore F_2 = \frac{-\mu_0}{4\pi} I_1 I_2 \oint_1 \oint_2 \frac{(dl_1 \cdot dl_2)(r_2 - r_1)}{|r_2 - r_1|^3} = -F_1$$

Using Biot and Savart's la...

exceed the velocity of light.

Ampere's law in magnetized material

$$J = J_b \times J_f$$

Ampere's law

$$\nabla \times B = \mu_0 J$$

$$\frac{1}{\mu_0}(\nabla \times B) = J_f + J_b$$

$$\frac{1}{\mu_0}(\nabla \times B) = J_f \times \cancel{\nabla \times M}$$

$$\frac{1}{\mu_0}(\nabla \times B) = J_f \times \nabla \times M.$$

$$\nabla \times \underbrace{\left(\frac{1}{\mu_0} B - M\right)}_{H} = J_f$$

$\nabla \times H = J_f$ Ampere's law in differential

$\cancel{\nabla \times H = J_f}$

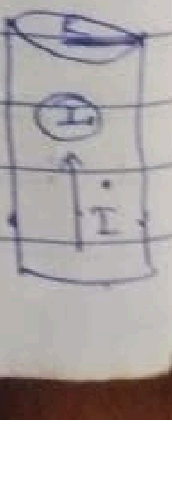
$$\int H \cdot dl = I_f \quad \text{(Integral form)}$$

Example:

Rod carrying uniformly distributed current I

$$\int H \cdot dl = I$$

$$H(2\pi r) = I\left(\frac{\pi r^2}{\pi R^2}\right)$$



Ampere's law: The magnetic field in space around an electric current is proportional to the electric current which serves as its source.

Question No.

Solution:

According to Ampere's force law the force on current element $I_1 dl_1$ of circuit 1 due to circuit 2 will be

$$dF_{12} = I_1 dl_1 \times B_2 \quad -(1)$$

But according to Biot Savart law

$$B_2 = \frac{\mu_0}{4\pi} \oint \frac{I_2 dl_2 \times r}{r^3} \quad -(2)$$

from eqn (1) and (2)

$$dF_{12} = \frac{\mu_0}{4\pi} I_1 I_2 \, dl_1 \times \oint_2 \frac{(dl_2 \times r)}{r^3} \quad -(3)$$

$$dF_{12} = \frac{\mu_0}{4\pi} I_1 I_2 \oint \frac{(dl_1 \cdot r) dl_2 - (dl_1 \cdot dl_2) r}{r^3} \quad -(a)$$

as $[\vec{A} \times (B \times C) = (A \cdot C) B - (A \cdot B) C]$

Hly

$$dF_{21} = I_2 dl_2 \times B_1 = I_2 dl_2 \times \oint_1 \frac{\frac{\mu_0}{4\pi} I_1 dl_1 \times (-r)}{r^3}$$

i.e) $dF_{21} = \frac{\mu_0}{4\pi} I_1 I_2 \oint_1 dl_2 \times \frac{(r \times dl_1)}{r^3}$

as $A \times B = -(B \times A)$

$$dF_{21} = \frac{\mu_0}{4\pi} I_1 I_2 \oint_1 \frac{(dl_2 \cdot dl_1) r - (dl_2 \cdot r) dl_1}{r^3} \quad -(b)$$

"The magnetic flux through the wire loop is proportional to the number of magnetic flux lines that pass through the loop."

When the flux through the surface changes, Faraday's law of induction says that the wire loop acquires an electromotive force (EMF).

The induced electromotive force in any closed circuit is equal to the rate of change of the magnetic flux enclosed by the circuit.

$$\mathcal{E} = -\frac{d\Phi_B}{dt},$$

where $\mathcal{E}$ is the EMF and Φ is the magnetic flux. The direction of the electromotive force is given by Lenz's law, which states that an induced current will flow in the direction that will oppose the change which produced it. This is due to the negative sign in

Inductance

From Faraday's law of induction, "any change in magnetic field through a circuit induces an electromotive force (EMF) (voltage) in the conductors, a process known as electromagnetic induction."

Inductance is defined as the ratio of the induced voltage to the rate of change of current causing it.

Faraday's law of induction makes use of the magnetic flux Φ_B through a region of space enclosed by a wire loop. The magnetic flux is defined by a surface integral.

$$\Phi_B = \int_{\Sigma} B \cdot dA.$$

Where dA is an element of the surface Σ enclosed by the wire loop. B is the magnetic field. The dot product $B \cdot dA$ corresponds to an infinitesimal amount of magnetic flux.

$$= \frac{1}{2}(\mu_0 n^2 A l) I^2$$

(2) [illegible]

were $U = \frac{1}{2} L I^2$

This equation gives the energy stored in the magnetic field of any inductor. We can see this by considering an arbitary inductor through which a changing current is passing. At any instant, the magnitude of the induced emf is $\epsilon = L\frac{di}{dt}$

where i is the induced current at that instance. Therefore, the power absorbed by the inductor is

$$p = \epsilon i = L\frac{di}{dt} i$$

The total energy stored in the magnetic field when the current increases from 0 to I. In a time interval from 0 to t can be determined by integrating this expression.

$$U = \int_0^t p\,dt' = \int_0^t L\frac{di}{dt'} i\,dt'$$

$$= L\int_0^t i\,di \quad \boxed{= \frac{1}{2} L I^2}$$

unit-III

Electromagnetic Induction

19/8/2020 (1)

Electromagnetic induction is the process a "current can be induced to flow due to a changing magnetic field".

: The force on a current - carrying wire due to the electrons wich move within it when a magnetic field is present is a classical example.

" This process also works in reverse. Either moving a wire through a magnetic field (or) (equivalently) changing the strength of the magnetic field over time can cause a current to flow.

1. Faraday's law

The rate of change of magnetic flux through a loop to the magnitude of the electro-motive force $\mathcal{E}$ induced in the loop. The relationship is

$$\mathcal{E} = \frac{d\Phi}{dt}$$

The electromotive force (or) EMF refers to the potential difference across the unloaded loop. (i.e) when the resistance in the circuit is high)

Energy in a magnetic fields.

$$\frac{dw}{dt} = -\xi I$$

$$= L I \frac{di}{dt}$$

$$w = \frac{1}{\ }$$

" Any change in magnetic field through a circuit produces an electromotive force (EMF) in the conductors, a process know as electromagnetic induction"!

Energy in a magnetic field. ①

2018)

The excitation of the space permeated by the magnetic field, it can be thought of as the potential energy that would be imparted on a charged particles moving through a region with an external magnetic field present.

The magnetic field component of an electro-magnetive wave carries a magnetic energy density U_B given by

$$U_B = \frac{B^2}{2\mu_0}.$$

where B is the amplitude of the magnetic field and $\mu_0 = 4\pi \times 10^{-7} \frac{N}{m^2}$ is the permeability of free space

we can assume that the magnetic field is essentially constant and given by

$B = \mu_0 n I$. everywhere inside the solenoid, Thus, the energy stored in a solenoid (or) the magnetic energy density times volume is equivalent

to $U = \mu_m(V) = \frac{(\mu_0 n I)^2}{2\mu_0}(Al)$.

Electrodynamics Before maxwell.

1. Gauss's Law

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

2. No magnetic monopole Law.

$$\nabla \cdot B = 0$$

3. Faraday's Law

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

4. Ampere's Law. $\nabla \times B = \mu_0 J$

Gauss' law states electric flux begins and ends on charge or at infinity.

The second law, which has no name, says magnetic field lines do not begin can end.

Faraday's laws states that a changing magnetic field produces an electric field.

Amper's law states magnetic fields are produced by moving charges.

$$\nabla \cdot J = -\nabla \cdot \varepsilon_0 \frac{\partial E}{\partial t}$$

Adding on this extra term gives maxwell's correction to Ampere's law.

Ampere's law now states.

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}.$$

maxwell did not use the continuity eqn. as a primary motive to arrive at the correction needed for Ampere's law.

— x —

Maxwell's Equations.

1. Gauss's Law. $\nabla \cdot E = \rho/\varepsilon_0$.
2. No magnetic Monopole Law. $\nabla \cdot B = 0$
3. Faraday's Law. $\nabla \times E = -\frac{\partial B}{\partial t}$.
4. Ampere's Law with displacement current

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}.$$

Unit - II

Boundary conditions

By Divergence Theorem

$$\int_V \nabla \cdot B \, d\tau = \int_S B \cdot \hat{e}_n ds \quad —(1)$$

The condition $\nabla \cdot B = 0$ is unaltered by the presence of magnetic materials. Therefore:

$$\int B \cdot \hat{e}_n ds = 0 \quad —(2)$$

This is Gauss' Theorem. The eqn means that the flux of the field B out of any closed surface is zero. Consider a small disc of height 'h' that the boundary between two media. If 'h' is very small ($h \to 0$) the integral $\int B \cdot \hat{e}_n ds = 0$ has contribution from the top and bottom ends only. i.e.

$$\int_1 B \cdot \hat{e}_n ds - \int_2 B \cdot \hat{e}_n ds = 0$$

Now $B \cdot \hat{e}_n ds = B_\perp ds$ where $B_\perp$ is the component of B normal to ds.

Therefore

$$\int_1 B \cdot \hat{e}_n ds = \int B(1)_\perp ds \text{ and } \int_2 B \cdot \hat{e}_n ds = -\int B(2)_\perp ds.$$

$$\oint \vec{H} \cdot d\vec{l} = I_{free\ enclosed}$$

$$\vec{B}' = \vec{B}\,(I_f + I_B)$$

In Electrostatics $\vec{E}$ is more useful than $\vec{D}$ ($\vec{B}$ does not depend on material)

$$\vec{H}' = \vec{H}\,(I_f)$$

$$\vec{\nabla} \cdot \vec{H} = -\vec{\nabla} \cdot \vec{M}$$

$$H = \frac{1}{\mu_0} B - M$$

$$\left(\vec{\nabla} \cdot \frac{1}{\mu_0} \vec{B}\right) = \vec{\nabla}(\vec{H} + \vec{M})$$

$$0 = \vec{\nabla} H + \vec{\nabla} \cdot \vec{M}$$

$$\vec{M} = X_m \vec{H}$$

X_m is the magnetic susceptibility.

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

$$= \mu_0(\vec{H} + X_m \vec{H})$$

$$= \mu_0 \vec{H}(1 + X_m)$$

$$\boxed{\vec{B} = \mu \vec{H}}$$

iron, nickel, cobalt, and their alloys

$$H = \frac{I}{2\pi R^2} r$$

$$H = \frac{I}{2\pi R^2} r\hat{\phi} \quad (r \le R)$$

$$H = \frac{I}{2\pi r} \phi \quad (r \geq R)$$

Outside the wire, $M = 0$, so $B = \mu_0 H$

$$= \frac{\mu_0 I}{2\pi r} \hat{\phi} \quad (r \geq R)$$

<u>Ex</u> B from a long solenoid with a magnetic material core

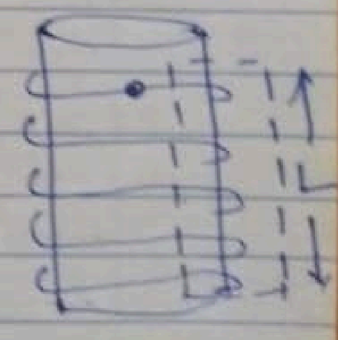
$$\int H \cdot dl = I$$

$HL = InL$ (n = # loops / length)

$H = In$.

inside solenoid $H = In\hat{z}$

outside solenoid $H = 0$

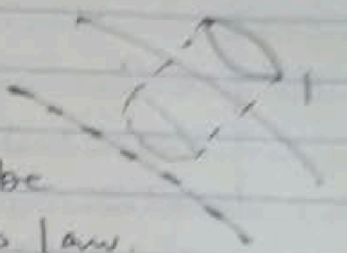


<u>Ampere's law</u>

For any closed loop path, the sum of the length elements times the magnetic field in the direction of the length element is equal to the permeability times the electric current enclosed in the loop.

(a) This eqn is true whatever the size of ds. Therefore.

$$B_{1\perp} = B_{2\perp}$$

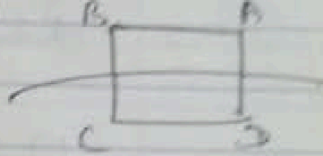


(c) $B_\perp$ is continuous.

The condition for H can be found by means of Ampere's law.

Consider a small circuit ABCDA in fig(2)

The sides BC, DA are extremely small and AB = CD = dl now



$$\oint H \cdot dl = I$$

where I is the current since BC, DA tend to zero.

$$H_1 dl - H_2 dl = I$$

where $H_{1\parallel}$, $H_{2\parallel}$ are the tangential components of H in the two media.

$$H_1 = H_2$$

It would be interesting to compare the boundary conditions in electrostatics and magnetostatics

1. The normal components of B is strictly continuous across the boundary. While the normal component of D is continuous only if there is no surface charge on
2. The tangential component of E is strictly continuous, while that of H is continuous across the boundary only if there is no surface current.

Question No.

Induced electric field.

An electric field is created in the conductor as a result of the changing magnetic flux. However this induced electric field has two important properties that distinguish it from the electrostatic field produced by stationary charges. The induced field is non conservative and can vary in time.

Neumann's formula

A formula for the mutual inductance M_{12} between two closed circuits C_1 and C_2 it is where 'r' is the distance between the elements ds_1 and ds_2. and μ_0 is the permeability of the empty space.

Energy in a magnetic field.

Energy stored in an inductor. As a result of the induced magnetic field inside an inductor of inductance L when a current i, flows through, energy is said to be stored in the magnetic field of the inductor.

Question No.

Magnetic susceptibility and permeability

In order to solve problems in magnetostatics it is essential to know relationship between B, H and M. These relationships depends on the nature of magnetic materials and are usually obtained from experiments.

(i) susceptibility χ_m

In large class of materials there exists an approximately linear relationship between M and H.

If the material is isotropic as well as linear

$$M = \chi_m H$$

i.e. $\chi_m = \frac{M}{H}$

where χ_m is a dimensionless scalar quantity and is called magnetic susceptibility

(ii) Permeability μ

A linear relationship between M and H implies also a linear relationship between B and H also

$B = \mu H$ i.e. $\mu = B/H$

Question No.

Ampere's law in magnetized materials

$$\vec{M} = \vec{M}(\vec{J_B}, \vec{K_B})$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{M}(\vec{r'}) \times (\vec{r'} - \vec{r})}{|\vec{r} - \vec{r'}|^3}\, d\tau'$$

$$\nabla' \frac{1}{|\vec{r} - \vec{r'}|} = \frac{\vec{r'} - \vec{r}}{|\vec{r} - \vec{r'}|^3}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\left[\int \frac{1}{|\vec{r} - \vec{r'}|}\left[\vec{\nabla}' \times \vec{M}(\vec{r'})\right] d\tau' + \oint \frac{1}{|\vec{r} - \vec{r'}|}\left[\vec{M}(\vec{r'}) \times d\vec{a'}\right]\right]$$

$$\vec{A}(\vec{r}) = \mu_0/4\pi\left[\int \frac{1}{|\vec{r} - \vec{r'}|}\, \vec{J}\, d\tau' + \oint \frac{1}{|\vec{r} - \vec{r'}|}\left[\cancel{M(\vec{r'}) \times d}\ \vec{K'} \cdot d\vec{a}\right]\right]$$

so,

$$\vec{J_B} = \vec{\nabla} \times M,$$

$$\vec{K_B} = \vec{M} \times \hat{n}$$

$$\vec{J} = \vec{J}_B + \vec{J}_{free}$$

$$\frac{1}{\mu_0}(\vec{\nabla} \times \vec{B}) = J = \vec{J_f} + \vec{\nabla} \times \vec{M}$$

$$\nabla \times \left(\frac{1}{\mu_0}\vec{B} - \vec{M}\right) = \vec{J_f}$$

let $\vec{H} = 1/\mu_0 \vec{B} - \vec{M}$;

$$\vec{\nabla} \times \vec{H} = \vec{J_f}$$

where μ is a scalar quantity and is called permeability. It measures the degree to which the specimen can be penetrated or permeated by the field H.

To find the relation between μ and χ_m definition of H. i.e.

$$H = (B/\mu_0) - M$$

or

$$B = \mu_0 (H + M)$$

$$\mu H = \mu_0 (H + \chi_m H)$$

(as $B = \mu H$ and $M = \chi_m H$)

$$\mu = \mu_0 (1 + \chi_m)$$

$$\mu/\mu_0 = 1 + \chi_m$$

$$\mu_r = 1 + \chi_m \text{ with } \mu_r = (\mu/\mu_0)$$

μ_r is called relative permeability.

<u>Problems</u> 1. Find the element of force dF_{12} caused by the current elements $I_2\,dl_2$ on $I_1\,dl$ as st also the force dF_{12} on the current element $I_2\,dl_2$ produced by $I_1\,dl_1$ show that they are unequal. How do you reconcile your results with Newton's third laws

Question No.

Let the change in flux linkage be

$$\phi = (\phi_2 - \phi_1)$$

So the change in flux linkage $N\phi$

Now the rate of change of flux linkage

$$\frac{N\phi}{t}$$

$$N\frac{d\phi}{dt}$$

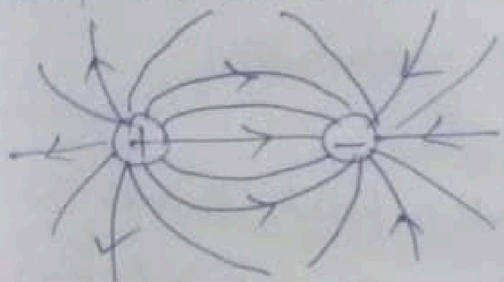
$$\boxed{E = N\frac{d\phi}{dt}}$$

Applications of Faraday's Law.

This law finds its application in most of the electrical machines, Industries and medical field etc.

1. Electrical Transformers.
2. The Basic working principle of electric generator is Faraday's law of mutual induction.
3. The Induction cooker,
4. Electro magnetic flow meter is used to measure velocity of certain fluids.
5. It is also used in musical instruments like electric guitar, electric violinele

coulomb electric field	Induced electric field
1. E is produced by charges.	1. E is produced by changing B, not by charges
2. coulomb electric field lines start / stop on charges.	2. Induced electric field lines form closed loop.
3. Induced E is conservative. $\mathcal{E} = \oint \vec{E} \cdot \vec{ds} = 0$	3. Induced E is non-conservative $\mathcal{E} = \oint \vec{E} \cdot \vec{ds} = -\frac{d\Phi_m}{dt}$



Faraday's law

$$\mathcal{E} = \oint \vec{E} \cdot \vec{ds} = -\frac{d\Phi_m}{dt}.$$

A changing magnetic field creates an induced electric field.

Lenz's law

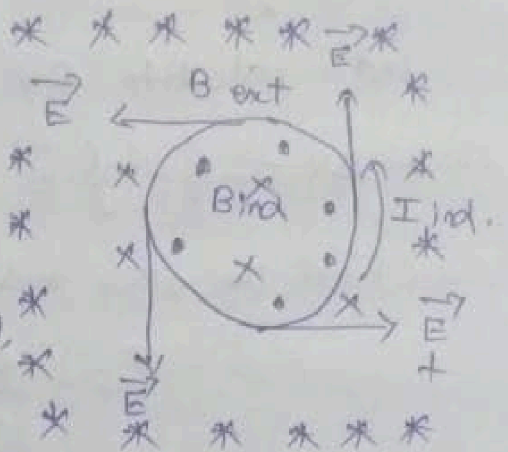
$$E = -N\left(\frac{d\Phi}{dt}\right) \text{ volts.}$$

Induced static field.

consider a conducting loop in an increasing uniform magnetic field,

$B_{ext} \nearrow \Rightarrow \Phi_B \nearrow \Rightarrow B_{ext} \uparrow\downarrow B_{ind} \Rightarrow$

I_{ind} CCW.



* So, according to Len's law, there is an induced current in the counter clockwise direction

* If there is a current, there must be something that acts on the charge carriers to make them move.

* So we infer there must be an induced electric field tangent to the loop at all points

* The conducting loop is not necessary to generate E

* The loop was used as a probe system to convince ourselves that there was the E field.

"The magnetic flux through the wire loop is proportional to the number of magnetic flux lines that pass through the loop."

When the flux through the surface changes, Faraday's law of induction says that the wire loop acquires an electromotive force (EMF).

The induced electromotive force in any closed circuit is equal to the rate of change of the magnetic flux enclosed by the circuit.

$$\mathcal{E} = -\frac{d\Phi_B}{dt},$$

where $\mathcal{E}$ is the EMF and Φ is the magnetic flux.

The direction of the electromotive force is given by Lenz's law, which states that an induced current will flow in the direction that will oppose the change which produced it. This is due to the negative sign in $\mathcal{E}$

$$\Delta V = V_f - V_i$$
$$= -\int_i^f \vec{E} \cdot \vec{ds}$$

let's calculate ΔV or $\mathcal{E}$ (emf) over the closed loop $\Delta V = \mathcal{E} = \oint \vec{E} \cdot \vec{ds}$

From the other side, Faraday's law:

$$\mathcal{E} = -\frac{d\Phi_m}{dt}$$

$$\oint \vec{E} \cdot \vec{ds} = -\frac{d\Phi_m}{dt}$$ This law implies that a changing magnetic flux will induce an induced electric field.

2. Two types of the electric field

when $A = \text{const}$ and $\theta = 0$.

$$\Phi_m = \vec{B} \cdot \vec{A} = BA \Longrightarrow \oint \vec{E} \cdot \vec{ds} = A \boxed{\frac{dB}{dt}}$$

6 Introduction to Maxwell's Equations. (4)

Maxwell's equations are a set of 4 complicated equations that describe the world of electromagnetics. These equation describe how electric and magnetic fields propagate, interact and how they are influenced by objects.

1. Gauss' Law

Gauss' Law is the first of maxwell's equations which dictates how the electric Field behaves around electric charges.

Gauss Law can be written in terms of the Electric flux density and the electric charge density as

$$\nabla \cdot D = \rho_V \quad \text{---} (1)$$

The symbol $\nabla \cdot$ is the divergence operator.

* Electric flux Density: The electric flux density (D) is related to the electric Field (E) by

$$D = \varepsilon E$$

"The amount of induced voltage is equal to the rate of change of the magnetic flux. This can be represented in eqn form.

$$\xi = \frac{\Delta \Phi}{\Delta t}$$

— x —

Q. Faraday's Laws of Electromagnetic Induction.

1. whenever the magnetic flux linked with a closed circuit changes, an induced e.m.f is set up the circuit whose magnitude at any instant is proportional to the rate of change of magnetic flux linked with the circuit.

If ξ is the magnetic flux linked with the circuit at any instant t and e is the induced e.m.f. Then

$$e \propto \frac{d\phi}{dt}.$$

(or)

$$\xi = -\frac{d\Phi_m}{dt}$$

Where L is the self inductance of a length l of the coaxial cable.

$$\frac{L}{l} = \frac{\mu_0}{2\pi} \ln \frac{R_2}{R_1}$$

(3)

(24) The definition of B and E in terms of the potentials A and Φ according to satisfies identically the two homogeneous maxwell's equations

The dynamic behaviour of A and Φ will be determined by the two inhomogeneous eqn.

In potential form.

$$\nabla^2\Phi + \frac{1}{c}\frac{\partial}{\partial t}(\nabla\cdot A) = -4\pi\rho$$

$$\nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} - \nabla\left(\nabla\cdot A + \frac{1}{c}\frac{\partial\Phi}{\partial t}\right) = -\frac{4\pi}{c}J$$

The vector potential is arbitrary to the extent that the gradient of some scalar function Λ can be added.

Thus B is left unchanged by the transformation. $A \to A' = A + \nabla\Lambda$.

In order to the electric field, the scalar potential must be simultaneously transformed

$$\Phi \to \Phi' = \Phi - \frac{1}{c}\frac{\partial \Lambda}{\partial t}.$$

a set of potentials (A, Φ) such that

$$\nabla\cdot A + \frac{1}{c}\frac{\partial\Phi}{\partial t} = 0.$$

(32)

The continuity eqn could be converted into a vanishing divergence by using Coulomb's law

Thus $\nabla \cdot J + \frac{\partial \rho}{\partial t} = \nabla \cdot \left(J + \frac{1}{4\pi} \frac{\partial D}{\partial t} \right) = 0$

Maxwell replaced J in Ampere's law by its generalisation

$$J \rightarrow J + \frac{1}{4\pi} \frac{\partial D}{\partial t}$$

for time-dependent fields. Thus Ampere's law become

$$\nabla \times H = \frac{4\pi}{c} J + \frac{1}{c} \frac{\partial D}{\partial t}$$

Maxwell called the added term in the displacement current.

The set of four equation

$$\left.\begin{aligned} &\nabla \cdot D = 4\pi \rho \\ &\nabla \cdot B = 0 \\ &\nabla \times H = \frac{4\pi}{c} J + \frac{1}{c} \frac{\partial D}{\partial t} \\ &\nabla \times E + \frac{1}{c} \frac{\partial B}{\partial t} = 0 \end{aligned}\right\}$$ known as

Maxwell's equations forms the basis of all electromagnetic phenomena.

(27) $\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} - \nabla\left(\nabla\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t}\right) = -\mu_0\vec{J}$

$$\boxed{\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -4\pi\rho}$$

$$\nabla^2\phi + \frac{\partial}{\partial t}(\nabla\cdot\vec{A}) = -\rho/\epsilon_0 \quad (8) \quad (or) = -4\pi\rho$$

$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} - \nabla\left(\nabla\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t}\right) = -\mu_0\vec{J} \quad (or) = -\frac{4\pi}{c}\vec{J} \quad (9)$$

maxwell eqns which is written in terms of the potentials.

Lorentz Gauge:

From the freedom of the transformation, a set of potentials $(\vec{A}, \phi)$ satisfy the Lorenz condition

$$\nabla\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t} = 0 \quad (10)$$

from eqn (8) & (9) it produces Inhomogeneous wave eqn.

$$\nabla^2\phi + \frac{\partial}{\partial t}\left(-\frac{1}{c^2}\frac{\partial\phi}{\partial t}\right) = \nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -\rho/\epsilon_0 \quad (11)$$

$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} - \nabla(0) = \nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2} = -\mu_0 J. \quad (12)$$

Displacement current:

~~In to~~ In terms of the rate of change of electric displacement field. Displacement current has ~~a~~ the units of electric current density, and it has as associated magnetic field just as ~~act~~ actual currents do.

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}.$$

In Electromagnetism, displacement current is a quantity appearing in maxwell's equations that is defined in terms of rate of change of electric displacement field.

If the current carrying wire passes certain symmetry, the magnetic field can be obtained by using Ampere's law

$$\oint B \cdot ds = \mu_0 \cdot I_{enclosed}.$$

The eqn states that line integral of magnetic field around the arbitrary closed loop is equal to $\mu_0 I_{enc}$, where I_{enc} is the conduction current passing through surface by closed path.

Vector and Scalar Potentials.

Maxwell's equations consists of a set of coupled first order partial differential equations relating the various components of electric and magnetic fields.

We used the scalar potential Φ and the vector potential A.

since $\nabla \cdot B = 0$ still holds,

we can define B in terms of a vector potential.

$$B = \nabla \times A$$

Faraday's law can be written as

$$\nabla \times \left(E + \frac{1}{c}\frac{\partial A}{\partial t}\right) = 0$$

This means that the quantity with vanishing curl in above eqn. can be written as the gradient of some scalar function, namely, a ~~sc~~ scalar potential Φ:

$$\left.\begin{aligned} E + \frac{1}{c}\frac{\partial A}{\partial t} &= -\nabla\Phi \\ E &= -\nabla\Phi - \frac{1}{c}\frac{\partial A}{\partial t} \end{aligned}\right\}$$

Maxwell's Displacement current : Maxwell's Equations

The basic laws of EMT which have discussed so far can be summarised in differential form by these four equations.

1. coulomb's Law : $\nabla . D = 4\pi \rho$

2. Ampere's Law : $\nabla \times H = \frac{4\pi}{c} J$

3 Faraday's Law : $\nabla \times E + \frac{1}{c}\frac{\partial B}{\partial t} = 0$

Absence of free magnetic poles : $\nabla . B = 0$

These equations are written in macroscopic form and in Gaussian Units.

It was derived for steady current phenomena with $\nabla . J = 0$. This requirement on the divergence of J is contained right in Ampere's law, as can be seen by taking the divergence of both sides

$$\frac{4\pi}{c} \nabla . J = \nabla . (\nabla \times H) = 0$$

while $\nabla . J = 0$ is valid for steady-state problems. The complete relation is is given by the continuity eqn for charge and current

$$\nabla . J + \frac{\partial \rho}{\partial t} = 0.$$

If the divergence of Ampere's law is taken an

$$\nabla \cdot (\nabla \times B) = \mu_0 (\nabla \cdot J)$$

The divergence of a curl is zero. The problem arises with the right side of the equation. In general, the right side is not zero.

For steady state currents, the divergence of J is zero. However outside of magnetostatics. Ampere's law cannot be right.

Maxwell's correction:

Ampere's law come from Biot-Savart which applies in the case of steady current. One motivation comes from the continuity equation and Gauss's law.

The continuity equation states.

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}.$$

Electric flux existing any volume that surrounds the charge, If there is negative charge within a volume, then there exists a negative amount of Electric flux existing). (9)

Gauss Law then is equivalent to the Force Equation for charges. which gives rise to the E field equation for point charges.

$$F = \frac{q_1 q_2}{4\pi \varepsilon_0 R^2}$$

$$|E| = \frac{q}{4\pi \varepsilon_0 R^2}$$

$$|D| = \frac{q}{4\pi R^2}$$

Gauss' Law means the following is true.

1. D and E field lines diverge away from positive charges
2. D and E field lines diverge towards negative charges
3. D and E field lines start and stop on Electric charges
4. Opposite charges attract and negative charges repel.

* Electric charge density:

$$\rho_V \quad \text{—(1)}.$$

The symbol in eqn(1) is the electric volume charge density. ρ is the electric charge and subscript V indicates it is the volume charge density.

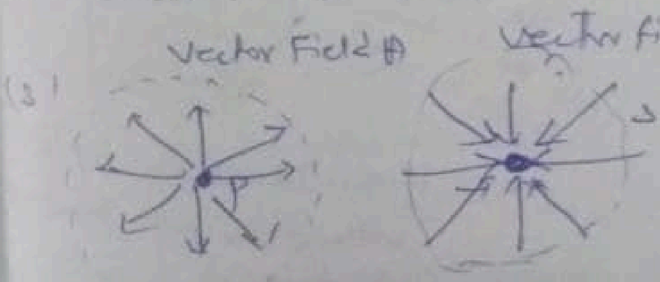
Ex

* Divergence Operator ∇.

Divergence at a point (x, y, z) is the measure of the vector flow out of a surface surrounding that point.

(e) Imagine a vector field represents water flow. Then if the divergence is a positive number, this means water is flowing out of the point (like a water spout).

If the divergence is a negative number, then water is flowing into the point (like a water drain - this location as sink).



(5)

Where ε is the permittivity of the medium (material) where we are measuring the fields.

The ~~elec~~ Electric Field is equal to the force per unit charge (at a distance R from a charge of value q_1

$$E = \frac{q_1}{4\pi \varepsilon R^2}$$

Then the Electric Flux density D

$$D = \varepsilon E = \frac{q_1}{4\pi R^2}$$

The Electric Flux Density D very similar to the Electric field, but does not depend on the material in which we are measuring in the permittivity ε.

D is field is a vector field. which means that every point in space it has a magnitude and direction. The Electric Flux Density has units of coulombs per meter squared (C/m^2).

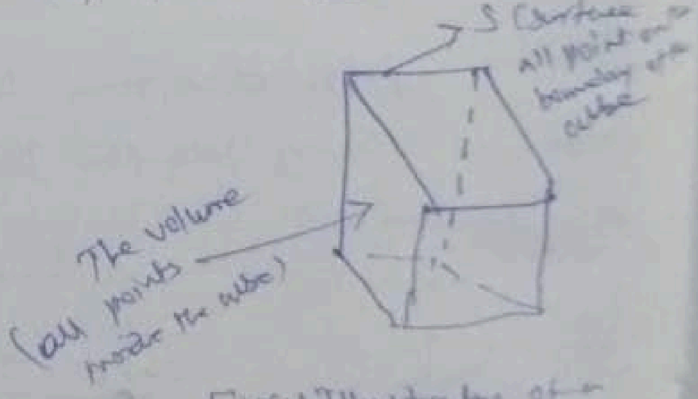
Equation ① is known as Gauss' Law in point form. If there exists electric charge somewhere, then the divergence of D at that point is nonzero, otherwise it is equal to zero.

Then integrating eqn ① over the volume V gives Gauss' Law in integral form.

$$\int_V (\nabla \cdot D)\, dv = \int_V \rho_v \, dv \quad \text{—②}$$

$$\Rightarrow \oint_S D \cdot ds = Q_{enc}.$$

We have a volume V which is the cube. The surface S is the boundary of the cube



Fig(1) Illustration of a volume V with boundary surface S.

5. The divergence of the D field over any region (volume) of space is exactly equal to the net amount of charge in that region. (10)

2. Gauss' Law for Magnetic Fields

$$\nabla \cdot D = \rho_v$$

$$\nabla \cdot B = 0$$

Gauss' Law for Electric Fields states that the divergence of the Electric flux density D is equal to the volume electric charge density. But

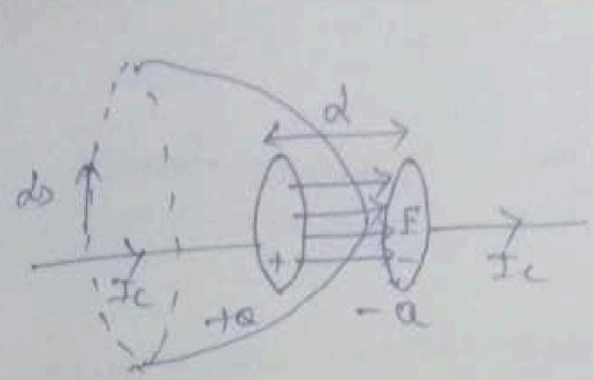
Gauss magnetism law states that the divergence of the magnetic flux density (B) is zero.

$$\nabla \cdot B = 0 \quad \text{—} \ (2)$$

$$\nabla \cdot H = 0 \quad (\text{since } B = \mu H)$$

since B and the magnetic field H are related by the permeability μ. eqn (2) that the divergence of the magnetic field is also zero.

III



we define the term on the right to be the displacement current $I_D = \epsilon_0 \frac{d\Phi_E}{dt}$

The changing electric flux through the curved surface is equivalent, in Amperes Law, to the current I_c through the flat surface. Thus we write a generalised form of Amperes law.

$$\oint \vec{B} \cdot \vec{ds} = \mu_0 (I_c + I_D)_{encl}$$

$$= \mu_0 I_{encl} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

magnetic fields are produced by both conduction currents and time varying electric fields.

=x=

(28) It can always find the potentials that satisfy the Lorentz condition

$$\nabla\cdot\vec{A}' + \frac{1}{c^2}\frac{\partial\phi'}{\partial t} = 0 \quad \text{--- (13)}$$

$$\nabla\cdot\vec{A}' + \frac{1}{c^2}\frac{\partial\phi'}{\partial t} = \nabla\cdot(\vec{A} + \nabla A) + \frac{1}{c^2}\frac{\partial}{\partial t}\left(\phi - \frac{\partial A}{\partial t}\right) \quad \text{--- (14)}$$

$$= \nabla\cdot\vec{A} + \nabla^2 A + \frac{1}{c^2}\frac{\partial\phi}{\partial t} - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = 0 \quad \text{--- (15)}$$

Thus, the gauge function A can be obtained to satisfy.

$$\nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = -\left(\nabla\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t}\right) \quad \text{--- (16)}$$

Here

$$\vec{A} - \vec{A}' + \nabla A$$

$$\phi \Longrightarrow \phi - \frac{\partial A}{\partial t}$$

$$\nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = 0$$

The potentials $\vec{A}$, ϕ and A in this restricted class are said to belong to the Lorentz gauge.

This will uncouple the pair of eqns. and leave two inhomogeneous wave eqns. one for Φ (25)

and one for A.

$$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -4\pi\rho$$

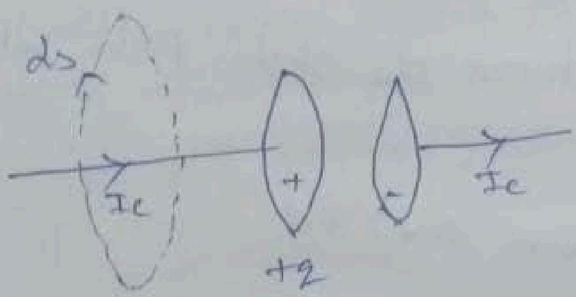
$$\nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = -\frac{4\pi}{c} J.$$

form a set of equations equivalent in all respects to maxwells equations.

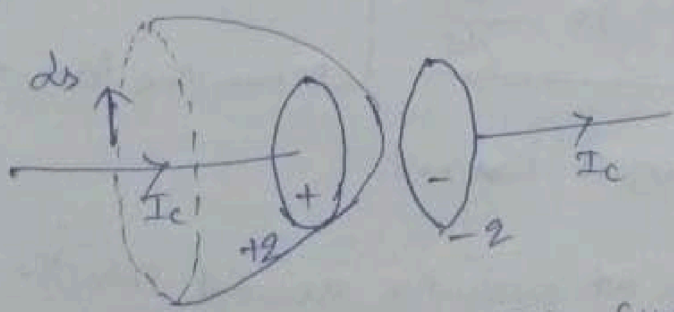
Derivation of Displacement Current:

Consider a charging capacitor. Conducting wires bring current I_c onto one plate and away from the other as the charge on the plates increases. For the path shown we can apply Ampere's Law and find.

$$\oint \vec{B} \cdot \vec{ds} = \mu_0 I$$
$$= \mu_0 I_c$$

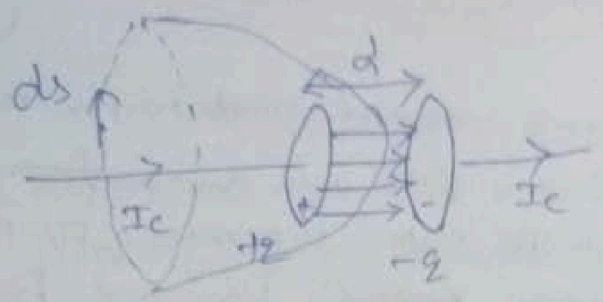


II. Consider a second surface that is also bounded by our path



Here no current pierces this surface since the charge stops on the capacitor plate. Hence

$$\oint \vec{B} \cdot \vec{ds} = \mu_0 I = 0$$



* The instantaneous charge on the capacitor plates is $q = C\Delta V$ where ΔV is the instantaneous potential difference across the plates.

* As capacitor charges the electric field between the plates is changing.

$$C = \varepsilon_0 \frac{A}{d} \text{ and } \Delta V = Ed$$

so $$q = C\Delta V = \varepsilon_0 A/d\,(Ed) = \varepsilon_0 EA = \varepsilon_0 \Phi_E$$

$$I_c = \frac{dq}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\boxed{I_D = \varepsilon_0 \frac{d\Phi_E}{dt}}$$

define the form of maxwell displacement current.

The net conduction current into the volume equals the net displacement current out of the volume. The generalised current is always continuous.

induced an opposite current on the right side.

magnetic Flux within a circuit produced an induced EMF, or voltage within the circuit,

$$EMF = -\frac{d\Phi}{dt} \quad \text{---} (2)$$

Φ is the magnetic Flux within a circuit. and EMF is the electromotive force.

$$\Phi(t) = \int_s B(t)\,ds$$

$$EMF_{total} = \oint_{circuit} d\,(EMF)$$

$$V = \int E.dl$$

$$E = \frac{dV}{dl}$$

$$EMF_{total} = \oint_{circuit} \vec{E}\,dL$$

$$\oint_{circuit} E.dL = \int_s \nabla \times E.ds$$

$$\text{Emf} = -\frac{d\phi}{dt}$$

$$\int_s \nabla \times E \cdot ds = -\frac{d}{dt}\int_s B(t)\cdot ds = \int_s -\frac{dB(t)}{dt}\cdot ds$$

$$\Rightarrow \boxed{\nabla \times E = \frac{-\partial B(t)}{\partial t}}$$

4. Ampere's Law.

$$\nabla \times H = \frac{\partial D}{\partial t} + J \quad \text{—} ①$$

$$\oint H \cdot dL = I_{enc} \quad \text{—} ②$$

$$\oint H \cdot dL = 2\pi r H = I_{enc}$$

$$\Rightarrow H = \frac{I_{enc}}{2\pi r}$$

$$H = \frac{I_{enc}}{2\pi r}$$

$$I_{enc} = \oint H \cdot dL = \int_s (\nabla \times H)\cdot ds$$

$$I_{enc} = \int_s J \cdot ds$$

$$\int_s (\nabla \times H)\cdot ds = \int_s J \cdot ds$$

$$\Rightarrow \nabla \times H = J$$

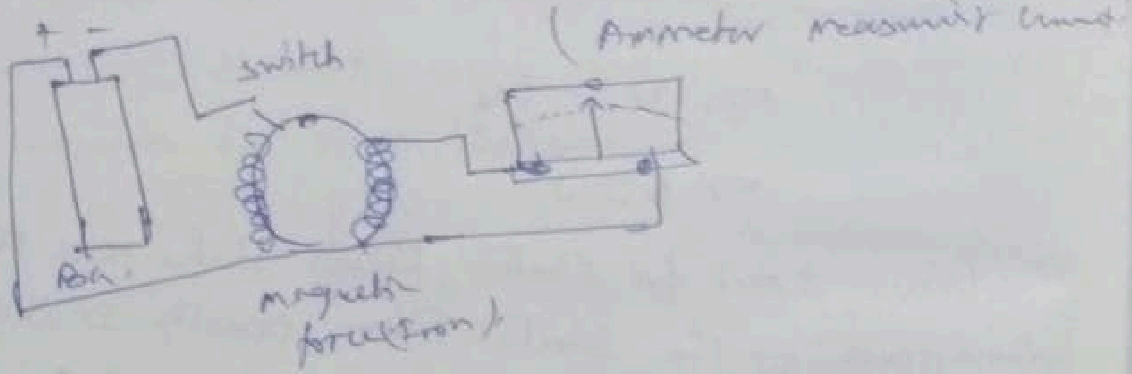
$$\nabla \times H = J$$

$$\nabla \cdot (\nabla \times H) = \nabla J$$

$$0 = \nabla \cdot J \quad //$$

3. Faraday's Law -

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad \text{—} (1)$$



The magnetic flux produced by the wired coil on the left exists within the wired coil on the right, which is connected to the ammeter.

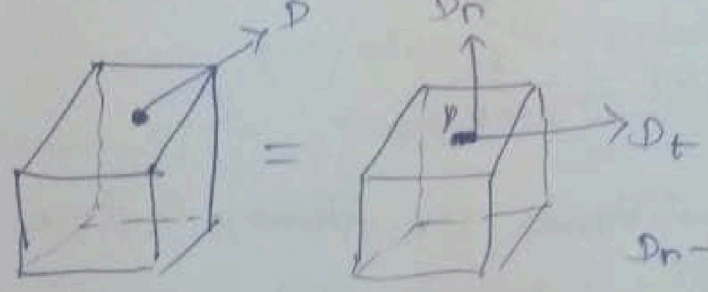
When the switch was initially changed from open to closed, the magnetic flux within the magnetic core increased from zero to some maximum number. When the flux was increasing, there existed an induced current on the opposite side.

Sly. When the switch was opened the magnetic flux in the core would decrease from it's constant value back to zero.

eqn ② states that the amount of charge inside a volume V ... $= V(= Q_{enc})$ is equal to the total amount of Electric Flux (Φ) exiting the surface S.
i.e) to determine the Electric Flux leaving the region V, we only need to know how much electric charge is within the volume. we rewrite eqn ① with more of the terms defined in eqn ②.

$D \cdot ds$.

$\int_S D \cdot ds$ This means we want to sum up the $D \cdot ds$ values at each point along the surface S.



D_n – Normal components

D_t – tangential components

Gauss' Law is a mathematical statement that the total Electric Flux exiting any volume is equal to the total charge inside. Hence, if the volume in question has no charge within it, the net flow of Electric Flux out of that region is zero.

If there is positive charge within a volume, then there exists a positive amount of

* Gauge Transformation.

$$\vec{A} \rightarrow \vec{A}' = \vec{A} + \nabla A \quad \text{— ①}$$

$$\phi \rightarrow \phi' = \phi - \frac{\partial A}{\partial t} \quad \text{— ②}$$

The invariance of the fields under gauge transformation is called gauge invariance.

consider to the vacuum form of the maxwell's eqn.

$$\nabla \cdot \vec{E} = \rho/\epsilon_0 \quad \text{— ③}$$

$$\nabla \cdot \vec{E} = -\nabla^2\phi - \frac{\partial}{\partial t}(\nabla \cdot \vec{A}) = \rho/\epsilon_0$$

$$\nabla^2\phi + \frac{\partial}{\partial t}(\nabla \cdot \vec{A}) = -\rho/\epsilon_0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0\epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{— ④}$$

$$\frac{\partial \vec{E}}{\partial t} = -\nabla \frac{\partial \phi}{\partial t} - \frac{\partial^2 \vec{A}}{\partial t^2} \quad \text{— ⑤}$$

$$\nabla \times \vec{B} = \nabla \times (\nabla \times \vec{A}) = -\nabla^2\vec{A} + \nabla(\nabla \cdot \vec{A}) \quad \text{— ⑥}$$

$$-\nabla^2 \cdot \vec{A} + \nabla(\nabla \cdot \vec{A}) = \mu_0 \vec{J} + \mu_0\epsilon_0\left(-\nabla\frac{\partial \phi}{\partial t} - \frac{\partial^2 \vec{A}}{\partial t^2}\right) \quad \text{— ⑦}$$

provided a gauge fuction A can be found to satisfy

$$\nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = -\left(\nabla A + 1/c \, \frac{\partial \phi}{\partial t}\right)$$

If we now employ the vector identity,

$$\nabla\cdot(E\times H) = H\cdot(\nabla\times E) - E\cdot(\nabla\times H) \quad \text{VIII}$$

and use Faraday's law. eqn ② becomes.

$$\int_V J\cdot E\, d^3x = -\frac{1}{4\pi}\int_V \left[c\nabla\cdot(E\times H) + E\cdot\frac{\partial D}{\partial t} + H\cdot\frac{\partial B}{\partial t}\right]d^3x \quad ③$$

We assume that the macroscopic medium involved is linear in its electric and magnetic properties.

if the total energy density is denoted by

$$u = \frac{1}{8\pi}(E\cdot D + B\cdot H) \quad ④$$

eqn ③ can be written as

$$-\int_V J\cdot E\, d^3x = \int_V \left[\frac{\partial u}{\partial t} + \frac{c}{4\pi}\nabla\cdot(E\times H)\right]d^3x \quad ⑤$$

Volume 'V' is a arbitrary, this can be cast into the form of a differential continuity eqn (or) conservation law,

$$\frac{\partial u}{\partial t} + \nabla\cdot S = -J\cdot E \quad ⑥$$

The vector S, representing energy flow is called Poynting vector, it is given by

$$S = \frac{c}{4\pi}(E\times H).$$

Poynting's Theorem

VII

considering for a single charge q the rate of doing work by external electromagnetic fields, E and B is $qV \cdot E$.

Where V is the velocity of the charge.

The magnetic field does no work, since the magnetic force is perpendicular to the velocity, If there exists a continuous distribution of charge and current, the total rate of doing work by the fields in a finite volume V' is $\int_V J \cdot E \, d^3x$ —①

This power represents a conservation of Electromagnetic energy into mechanical (or) thermal energy.

$$\int_V J \cdot E \, d^3x = \frac{1}{4\pi} \int_V \left[cE \cdot (\nabla \times H) - E \cdot \frac{\partial D}{\partial t} \right] d^3x$$ —②

3. The divergence of the D field over any region (volume) of space is exactly equal to the net amount of charge in that region. (10)

2. Gauss' Law for Magnetic Fields

$$\nabla \cdot D = \rho_v$$

$$\nabla \cdot B = 0$$

Gauss' Law for Electric Fields states that the divergence of the Electric flux density D is equal to the volume electric charge density. But Gauss magnetism law states that the divergence of the magnetic flux density (B) is zero.

$$\nabla \cdot B = 0 \quad \text{—— ②}$$

$\nabla \cdot H = 0$ (since $B = \mu H$)

Since B and the magnetic field H are related by the permeability μ. eqn ② that the divergence of the magnetic field is also zero.

curl E

curl means in simple manner.

06/8/20

$\nabla \times E = 0$

The curl of electric field is not always zero. The general law for the curl of the electric field is one of the maxwell's equations which states that

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ It is only when the magnetic field is either zero (or) constant so it true that the curl of electric field is zero.

Now, Electric field due to a point charge at a distance r is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

consider the point charge is placed at the centre on it origin.

Now in spherical co-ordinates system

$$\vec{dl} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$$

Now $\vec{E}\cdot\vec{dl} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}dr.$

N1

The term $\int_V \frac{du_{em}}{dt} dV$ (or)

$$\frac{\partial}{\partial t}\int_V \left[\frac{1}{2}\mu H^2 + \frac{1}{2}\epsilon E^2\right] dV$$ the terms

$\frac{1}{2}\mu H^2$ and $\frac{1}{2}\epsilon E^2$ represent the energy stored in electric and magnetic fields respectively and their sum denotes the total energy stored in electromagnetic field. So total terms gives the rate of decrease of energy stored in volume V due to electric and magnetic fields.

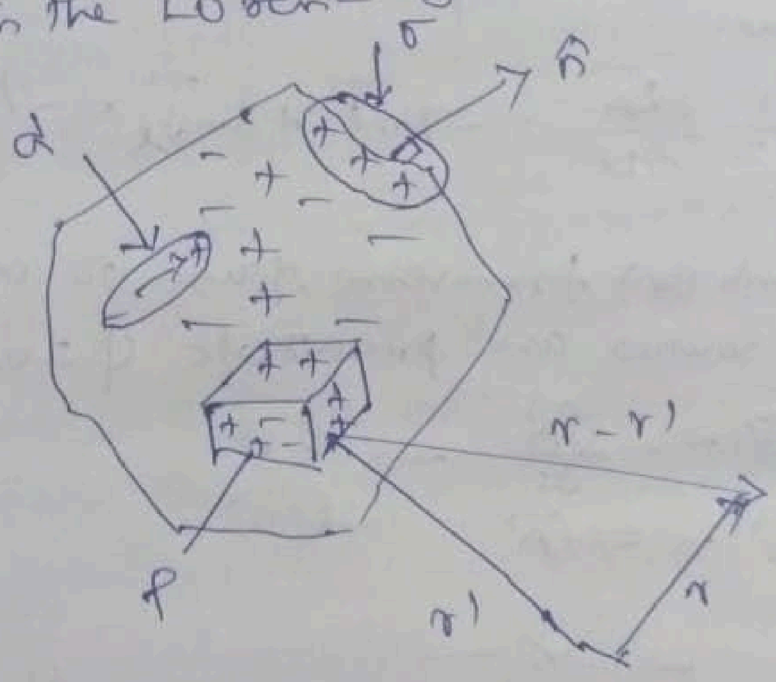
$\int_V (\vec{E}\cdot\vec{J})\,dV$ gives the rate of energy transferred into the electromagnetic field.

This is also known as work - energy theorem. This is also called as the energy conservation law in electromagnetism.

Retarded potential.

The retarded potentials are the electromagnetic potentials for the electromagnetic field generated by time varying electric current (or) charge distributions.

In the Lorenz gauge :-



Position vectors r and r' used in the calculation.

The starting point is maxwell's eqn in the potential formulation using the Lorenz Gauge

$$\Box\varphi = \rho/\epsilon_0 \, , \quad \Box A = \mu_0 J$$

III

i.e) $\vec{H}\,\text{curl}\,\vec{E} = -\vec{H}\cdot\frac{\partial\vec{B}}{\partial t}$ — (5)

and $\vec{E}\cdot\text{curl}\,\vec{H} = \vec{E}\vec{J} + \vec{E}\frac{\partial\vec{D}}{\partial t}$ — (6)

doing (6) - (5) i.e)

$$\vec{H}\cdot\text{curl}\,\vec{E} - \vec{E}\cdot\text{curl}\,\vec{H} = -H\frac{\partial\vec{B}}{\partial t} - \vec{E}\vec{J} - E\frac{\partial\vec{D}}{\partial t}$$

$$= -\left[H\frac{\partial\vec{B}}{\partial t} + \vec{E}\cdot\frac{\partial\vec{D}}{\partial t}\right] - \vec{E}\cdot\vec{J}$$

as $\text{div}\,(\vec{A}\times\vec{B}) = \vec{B}\,\text{curl}\,\vec{A} - \vec{A}\,\text{curl}\,\vec{B}$

So $$\vec{H}\cdot\frac{\partial\vec{B}}{\partial t} = -\left[\vec{H}\cdot\frac{\partial\vec{B}}{\partial t} + \vec{E}\frac{\partial\vec{D}}{\partial t}\right] - \vec{E}\cdot\vec{J} \quad (7)$$

But $$\vec{B} = \mu\vec{H} \text{ and } \vec{D} = \epsilon\vec{E}$$

so, $$\vec{H}\cdot\frac{\partial\vec{B}}{\partial t} = \vec{H}\cdot\frac{\partial}{\partial t}(\mu\vec{H})$$

$$= \tfrac{1}{2}\mu\frac{\partial}{\partial t}(H^2)$$

$$= \frac{\partial}{\partial t}\left[\tfrac{1}{2}\vec{H}\cdot\vec{B}\right]$$

and $$\vec{E}\cdot\frac{\partial\vec{D}}{\partial t} = E\cdot\frac{\partial}{\partial t}(\epsilon\vec{E})$$

$$= \tfrac{1}{2}\epsilon\frac{\partial}{\partial t}(E)^2$$

$$= \frac{\partial}{\partial t}\left[\tfrac{1}{2}\vec{E}\cdot\vec{D}\right]$$

using the continuity eqn

$$\nabla . \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\frac{1}{c^2} \nabla \frac{\partial \phi}{\partial t} = \mu_0 \frac{1}{4\pi} \nabla \int \frac{-\nabla' . \vec{J}}{|\vec{x}' - \vec{x}'|} d^3x' \quad -(25)$$

$$= \mu_0 \left(-\frac{1}{4\pi} \nabla \int \frac{\nabla' . \vec{J}}{|\vec{x}' - \vec{x}''|} d^3x' \right) = \mu_0 \vec{J}_l \quad -(26)$$

eqn (22) becomes.

$$\nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J} + \mu_0 \vec{J}_l = -\mu_0 \vec{J}_t \quad -(27)$$

The coulomb (or) transverse gauge is often used when no sources are present ie $\phi = 0$.

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} \quad -(28)$$

$$\vec{B} = \nabla \times \vec{A} \quad -(29)$$

— x —

Poynting vector and Poynting theorem. I

When electro magnetic wave travels in space, it carries energy and energy density is always associated with electric fields and magnetic fields.

The rate of energy travelled through per unit area i.e) The amount of energy flowing through per unit area in the perpendicular direction to the incident energy per unit time is called poynting vector.

Mathematically poynting vector is represented as $\vec{P} = \vec{E} \times \vec{H} \left(= \frac{\vec{E} \times \vec{B}}{\mu} \right)$

The direction of poynting vector is perpendicular to the plane containing $\vec{E}$ and $\vec{H}$. Poynting vector is also called as instantaneous ~~energ~~ energy flux density. Here rate of energy transfer $\vec{P}$ is perpendicular to both $\vec{E}$ and $\vec{H}$. Since it represents the rate of energy transfer per unit area. its units is W/m^2

$$\int_S (\vec{E} \times \vec{H}) \cdot d\vec{s} = -\int_V \frac{\partial u_{em}}{\partial t} dv - \int_V (\vec{E} \cdot \vec{J}) dv \quad \underline{V}$$

$$\int_S \vec{P} \cdot d\vec{s} = -\int_V \frac{\partial u_{em}}{\partial t} dv - \int_V (\vec{E} \cdot \vec{J}) dv$$

$$(\text{as } \vec{P} = \vec{E} \times \vec{H}) \quad \text{—— (9)}$$

i.e) Total power leaving the volume = rate of decrease of stored e.m. energy — Ohmic power dissipated due to charge motion.

The eqn (9) represents the poynting theorem according to which the net power flowing out of a given volume is equal to the rate of decrease of stored electromagnetic energy in that volume minus the conduction losses.

In eqn (9) $\int_S \vec{P} \cdot d\vec{s}$ represents the amount of electromagnetic energy crossing the closed surface per second (or) the rate of flow of outward energy through the surface S enclosing volume V i.e) it is poynting vector.

so from eqn (7)

$$\operatorname{div}(\vec{E}\times\vec{H}) = -\frac{\partial}{\partial t}\left[\tfrac{1}{2}(\vec{H}\cdot\vec{B}+\vec{E}\cdot\vec{D})\right] - \vec{E}\cdot\vec{J}$$

(or)

$$\vec{E}\cdot\vec{J} = -\frac{\partial}{\partial t}\left[\tfrac{1}{2}(\vec{H}\cdot\vec{B}+\vec{E}\cdot\vec{D})\right] - \operatorname{div}(\vec{E}\times\vec{H})$$

integrating eqn (8) over a volume V enclosed by a surface S,

$$\int_V \vec{E}\cdot\vec{J}\,dv = -\int_V \left|\frac{\partial}{\partial t}\left\{\tfrac{1}{2}(\vec{H}\cdot\vec{B}+\vec{E}\cdot\vec{D})\right\}\right| dv - \int_V \operatorname{div}(\vec{E}\times\vec{H})\,dv$$

$$\int_V \vec{E}\cdot\vec{J}\,dv = -\int_V \left[\tfrac{1}{2}\mu H^2 + \tfrac{1}{2}\epsilon E^2\right] dv - \int_S (\vec{E}\times\vec{H})\,ds$$

as $\vec{B}=\mu\vec{H}$, $\vec{D}=\epsilon\vec{E}$ and

$$\int_V \operatorname{div}(\vec{E}\times\vec{H})\,dv = \int_S (\vec{E}\times\vec{H})\cdot ds$$

(or)

$$\int_V (\vec{E}\cdot\vec{J})\,dv = -\frac{\partial}{\partial t}\int_V \left[\tfrac{1}{2}\mu H^2 + \tfrac{1}{2}\epsilon E^2\right] dv \cdot \int_S (\vec{E}\times\vec{H})\cdot ds$$

(84)

$$-E = \nabla\phi + \frac{\partial A}{\partial t}$$

$$B = \nabla \times A$$

the advanced time

$$t_a = t + \frac{|r - r'|}{c}$$

replaces the retarded time.

In the case the fields are time-independent the time derivatives in the $\Box$ operators of the fields are zero, and maxwell's eqns reduce to

$$\nabla^2\phi = -\rho/\epsilon_0, \quad \nabla^2 A = -\mu_0 J$$

where ∇^2 is the laplacian. which take the form of poisson's eqn. in four components (one for ϕ and three for A) and the solutions

$$\phi(r) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')}{|r-r'|} d^3r'$$

$$A(r) = \frac{\mu_0}{4\pi}\int \frac{J(r')}{|r-r'|} d^3r'.$$

(33) where $\phi(r,t)$ is the electric potential and $A(r,t)$ is the magnetic vector potential, for an arbitrary source of charge density $\rho(r,t)$ and current density $J(r,t)$ and $\Box$ is the D'Alembert operator.

Solving these gives the retarded potentials below

For time-dependent fields.

$$\phi(r,t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r', t_r)}{|r-r'|} \, d^3r'$$

$$A(r,t) = \frac{\mu_0}{4\pi} \int \frac{J(r', t_r)}{|r-r'|} \, d^3r'.$$

where r is a point in space, t is time,

$$t_r = t - \frac{|r-r'|}{c}$$

is the retarded time, and d^3r' is the integration measure using r'.

From $\phi(r,t)$ and $A(r,t)$ the fields $E(r,t)$ and $B(r,t)$ can be calculated using the definitions of the potentials.

Coulomb Gauge

The coulomb, radiation (or) transverse gauge is

$$\nabla \cdot \vec{A} = 0$$

we obtain the poisson's eqn again.

$$\nabla^2 \phi + \frac{\partial}{\partial t}(0) = \nabla^2 \phi = -\rho/\epsilon_0 \quad \text{--- (17)}$$

And solution is very well-known.

$$\phi(\vec{x}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{x}', t)}{|\vec{x} - \vec{x}'|} d^3x' \quad \text{--- (18)}$$

From eqn (9)

The vector potential satisfies the inhomogeneous wave eqn.

$$\nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} - \nabla\left(0 + \frac{1}{c^2}\frac{\partial \phi}{\partial t}\right) = -\mu_0 \vec{J} \quad \text{--- (19)}$$

$$\nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J} + \nabla \frac{1}{c^2}\frac{\partial \phi}{\partial t} \quad \text{--- (20)}$$

Current density $\vec{J}$ can be decomposed into two pieces a longitudinal (irrotational), $\vec{J}_l$ and transverse (rotation) J_t:

$$\vec{J} = \vec{J}_l + \vec{J}_t$$

(39)

$$\nabla \times \vec{J}_l = 0$$
$$\nabla \cdot \vec{J}_t = 0$$

The vector identity

$$\nabla \times (\nabla \times \vec{J}) = \nabla(\nabla\cdot\vec{J}) - \nabla^2\vec{J}$$

From this identity, the current density splits into two parts:

$$\nabla^2(\vec{J}_l + \vec{J}_t) = \nabla\times(\nabla\times\vec{J}) - \nabla(\nabla\cdot\vec{J}) \quad \text{--- (21)}$$
$$= \nabla^2\vec{J}_t = \nabla\times(\nabla\times\vec{J}) \quad \text{--- (22)}$$
$$\nabla^2\vec{J}_l = -\nabla(\nabla\cdot\vec{J}) \quad \text{--- (23)}$$

Eqns (22) & (23) are poisson eqn. and we have the form $\nabla^2\left(\frac{1}{|\vec{x}-\vec{x}'|}\right) = -4\pi\delta(\vec{x}-\vec{x}')$

$$\vec{J}_t = \frac{1}{4\pi}\nabla\times\nabla\times\int\frac{\vec{J}}{|\vec{x}-\vec{x}'|}d^3x'$$

$$\vec{J}_l = -\frac{1}{4\pi}\nabla\int\frac{\nabla'\cdot\vec{J}}{|\vec{x}-\vec{x}'|}d^3x'$$

Then $\frac{1}{c^2}\nabla\frac{\partial\phi}{\partial t} = \frac{1}{c^2}\frac{1}{4\pi\epsilon_0}\nabla\int\frac{\partial}{\partial t}\frac{\rho(\vec{x}',t)}{|\vec{x}-\vec{x}'|}d^3x'$

$$= \mu_0\epsilon_0\frac{1}{4\pi\epsilon_0}\nabla\int\frac{\partial\rho(\vec{x},t)}{\partial t}\frac{1}{|\vec{x}-\vec{x}'|}d^3x' \quad \text{--- (24)}$$

Now $\int_a^b \vec{E}\cdot d\vec{l} = \frac{1}{4\pi\varepsilon_0} q \int_a^b \frac{dr}{r^2}$

$$= \frac{1}{4\pi\varepsilon_0} q \left(\frac{-1}{r}\right)_{ra}^{rb}$$

$$\int_a^b \vec{E}\cdot d\vec{l} = \frac{1}{4\pi\varepsilon_0} q \left(-\frac{1}{rb} + \frac{1}{ra}\right)$$

$$\int_a^b \vec{E}\cdot d\vec{l} = \frac{1}{4\pi\varepsilon_0} q \left(\frac{1}{ra} - \frac{1}{rb}\right)$$

When $ra = rb$.

$$\oint \vec{E}\cdot d\vec{l} = \frac{1}{4\pi\varepsilon_0} \frac{q}{ra} \underset{(0)}{\cancel{(1-1)}}$$

$\Rightarrow \oint \vec{E}\cdot d\vec{l} = 0$.

by Stokes theorem, $\oint (\vec{\nabla}\times\vec{E})\, dv = \oint \vec{E}\cdot d\vec{l} = 0$

$$\boxed{\vec{\nabla}\times\vec{E} = 0}$$

Curl of E is zero. //.

Potential field.

The Laplace equation is equivalent in three dimensions to the inverse square law of gravitational (or) electrical attraction (in source free regions: in regions with sources, it becomes Poisson's eqn). Ex. of potential fields include the field of the gravity, potential, and static electric and magnetic fields.

Gravity: The earth's gravitational field (or) the attractive force produced by the mass of the Earth. variations in the gravitational field can be used to map changes in the density of formations in the Earth.

Laplace eqn: A partial differential eqn that governs potential fields and is equivalent, in three dimensions, to the inverse square law of gravitational (or) electrical attraction. In cartesian coordinates the Laplace eqn equates the sum of the second partial derivatives of the field to zero.

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

where $u(x,y,z)$ is a potential function.

— x —